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THE

ARITHMETIC OF ELECTRICITY

A MANUAL OF ELECTRICAL CALCULATIONS
BY ARITHMETICAL METHODS

Including numerous rules, examples, and tables in the field of practical electrical engineering and experimenting.

BY

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ILLUSTRATED

NEW YORK
NORMAN W. HENLEY & CO
150 NASSAU STREET
LONDON
E. & F. N. SPON
1891

Mrs. H. L. [illegible]
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PREFACE.

The solution of a problem by arithmetic, although in some cases more laborious than the algebraic method, gives the better comprehension of the subject. Arithmetic is analysis and bears the same relation to algebra that plane geometry does to analytical geometry. Its power is comparatively limited, but it is exceedingly instructive in its treatment of questions to which it applies.

In the following work the problems of electrical engineering and practical operations are investigated on an arithmetical basis. It is believed that such treatment gives the work actual value in the analytical sense, as it necessitates an explanation of each problem, while the adaptability of arithmetic to readers who do not care to use algebra will make this volume more widely available.

In electricity there is much debatable ground, which has been as far as possible avoided. Some points seem quite outside of the scope of this book, such as the introduction of the time-constant in battery calculations. Again the variation in con-

stants as determined by different authorities made a selection embarassing. It is believed that some success has been attained in overcoming or compromising difficulties such as those suggested.

Enough tables have been introduced to fill the limits of the subject as here treated.

The full development of electrical laws involves the higher mathematics. One who would keep up with the progress of the day in theory has a severe course of study before him. In practical work it is believed that such a volume as the Arithmetic of Electricity will always have a place. We hope that it will be favorably received by our readers and that their indulgence will give it a more extended field of usefulness than it can pretend to deserve.

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ARITHMETIC OF ELECTRICITY.

CHAPTER I.

INTRODUCTORY.

SPACE is the lineal distance from one point to another.

Time is the measure of duration.

Force is any cause of change of motion of matter. It is expressed practically by grams, volts, pounds or other unit.

Resistance is a counter-force or whatever opposes the action of a force.

Work is force exercised in traversing a space against a resistance or counter-force. Force multiplied by space denotes work as foot-pounds.

Energy is the capacity for doing work and is measurable by the work units.

Mass is quantity of matter.

Weight is the force apparent when gravity acts upon mass. When the latter is prevented from moving under the stress of gravity its weight can be appreciated.

Physical and Mechanical calculation, are based on three fundamental units of dimension, as follows: the unit of time—the second, T; the unit of length —the centimeter, L; the unit of mass—the gram, M. Concerning the latter it is to be distinguished from weight. The gram is equal to one cubic centimeter of water under standard conditions and is invariable; the weight of a gram varies slightly with the latitude and with other conditions.

Upon these three fundamental units are based the derived units, geometrical, mechanical and electrical. The derived units are named from the initials of their units of dimension, the C. G. S. units, indicating centimeter-gram-second units.

In practical electric calculations we deal with certain quantities selected as of convenient size and as bearing an easily defined relation to the fundamental units. They are called practical units.

The cause of a manifestation of energy is force; if of electromotive energy, that is to say of electric energy in the current form, it is called electromotive force, E. M. F. or simply E. or difference of potential D. P. What this condition of excitation may be is a profound mystery, like gravitation and much else in the physical world. The practical unit of E. M. F. is the VOLT, equal to one hundred millions (100,000,000) C. G. S. units of E. M. F. The last numeral is expressed more briefly as the eighth

Volt = practical unit of electromotive force. It is 10^8 absolute units of E.M.F. in C.G.S. system, and is a little less than the E.M.F. of a Daniell cell.

Ampere (Am-pâr) = unit of strength of electric current. It is the current which flows through a conductor whose resistance is one Ohm, and between the two ends of which the unit difference of potential, one volt, is maintained.

Coulomb (kö-lom') = unit of quantity of current of electricity = quantity furnished by a current of one ampere in one second.

power of 10 or 10^8. Thus the volt is defined as equal to 10^8 C. G. S. units of E. M. F.

This notation in powers of 10 is used throughout C. G. S. calculations. Division by a power of 10 is expressed by using a negative exponent, thus 10^{-8} means $\frac{1}{100000000}$. *The exponent indicates the number of ciphers to be placed after* 1.

When electromotive force does work a current is produced. The practical unit of current is the AMPERE, equal to $\frac{1}{10}$ C. G. S. unit, or 10^{-1} C. G. S. unit, $\frac{1}{10}$ being expressed by 10^{-1}.

A current of one ampere passing for one second gives a quantity of electricity. It is called the COULOMB and is equal to 10^{-1} C. G. S. units.

A coulomb of electricty if stored in a recipient tends to escape with a definite E. M. F. If the recipient is of such character that this definite E. M. F. is one volt, it has a capacity of one FARAD equal to $\frac{1}{1000000000}$ or 10^{-9} C. G. S. unit.

A current of electricity passes through some substances more easily than through others. The relative ease of passage is termed conductance. In calculations its reciprocal, which is resistance, is almost universally used. A current of one ampere is maintained by one volt through a resistance of one practical unit. This unit is called the OHM and is equal to 10^9 C. G. S. units.

Sometimes, where larger units are wanted, the prefix *deka*, ten times, *heka*, one hundred times, *kilo*, one

Farad. Electromotive ...

British Assn in 1863, & called the B. A. unit of resistance. The legal ohm adopted by the Electric Congress of 1884 = resistance at 0° C. of a column of mercury of 1 sq. millimeter, and 106. centimeters long. The Siemens unit is the resistance of a like column 1 meter long. The resistance of a copper wire 1/10" in diameter and 1000 ft. long is very nearly one ohm. A mile of ordinary telegraph wire has a resistance of about 13 ohms

thousand times, or *mega,* one million times are used, as *dekalitre,* ten liters, *kilowatt,* one thousand watts, *megohm,* one million ohms.

Sometimes, where smaller units are wanted, the prefixes, *deci,* one tenth, *centi,* one hundredth, *milli,* one thousandth, *micro,* one millionth, are used. A microfarad is one millionth of a farad.

For the concrete conception of the principal units the following data are submitted.

A Daniell's battery maintains an E. M. F. of 1.07 *volt.* A current which in each second deposits
1/1000 = .00033 grams copper (by electro-plating) is of one *ampere* intensity and from what has been said the copper deposited by that current in one second corresponds to one *coulomb.* A column of mercury one millimeter square and 106.24 centimeters long has a resistance of one *ohm* at 0° C. The capacity of the
1/572 = earth is $\frac{686}{1000000}$ *farad.* A Leyden jar with a total coated surface of one square meter and glass one mm. thick has a capacity of $\frac{1}{55}$ *microfarad.* The last is the more generally used unit of capacity.

These practical units are derived from the C. G. S. units by substituting for the centimeter (C.) one thousand million (10^9) centimeters and for the gram, the one hundred thousand millionth (10^{-11}) part of a gram.

Dyne = unit of force in C. G. S. system = that force which acting on one gram for one second generates a velocity of one centimeter per sec. = 1/g. x 1/1000 = 1 per milligram; or [illegible] dynes = 1 lb. at the earth's surface. [illegible] Calculate. A megadyne = 1.02 Kilogram

Erg = Amount of work done by the unit of force [illegible] acting through [illegible] the [illegible] of distance one centimeter. One ft. pound = 1.356 x 10⁷ [illegible]

[redacted]

Watt = practical unit of electrical activity or power = 10^7 ergs per second, or the same number of absolute C.G.S. units of electrical activity; or it is the rate of working in a circuit when the E.M.F. is one volt and the current one ampere. One horse power = 746 watts.

Horse power = the measure of the rate at which a horse works in drawing, or a prime motor works, = 550 ft. pounds per second or 7460 megaergs per second. The real power of a horse is about 3/4 of the horse power as here given.

CHAPTER II.

OHM'S LAW.

THIS law expresses the relation in an active electric circuit (circuit through which a current of electricity is forced) of current, electromotive force, and resistance. These three factors are always present in such a circuit. Its general statement is as follows :

In an active electric circuit the current is equal to the electromotive force divided by the resistance.

This law can be expressed in various ways as it is transposed. It may be given as a group of rules, to be referred to under the general title of OHM'S LAW.

Rule 1. The current is equal to the electromotive force divided by the resistance. $C = \frac{E}{R}$ = 10^{-1} = $\frac{10^8}{10^9}$ C.G.S. Ampere Volt Ohm

Rule 2. The electromotive force is equal to the current multiplied by the resistance. $E = C\,R$

Rule 3. The resistance is equal to the electromotive force divided by the current. $R = \frac{E}{C}$

Rule 4. The current varies directly with the electromotive force and inversely with the resistance.

Rule 5. The resistance varies directly with the electromotive force and inversely with the current.

Rule 6. The electromotive force varies directly with the current and with the resistance.

This law is the fundamental principle in most electric calculations. If thoroughly understood it will apply in some shape to almost all engineering problems. The forms 1, 2, and 3 are applicable to integral or single conductor circuits; when two or more circuits are to be compared the 4th, 5th and 6th are useful. The law will be illustrated by examples.

SINGLE CONDUCTOR CLOSED CIRCUITS.

These are circuits embracing a continuous conducting path with a source of electromotive force included in it and hence with a current continually circulating through them.

EXAMPLES.

A battery of resistance 3 ohms and E. M. F. 1.07 volts sends a current through a line of wire of 55 ohms resistance; what is the current?

Solution: The resistance is $3 + 55 = 58$ ohms. By rule 1 we have for the current $\frac{1.07}{58}$ giving .01845 Ampere.

Note.—A point to be noticed here is that whatever is included in a circuit forms a portion of it and its resistance must be included therein. Hence the resistance of the battery has to be taken into account. The resistance of a battery or generator is sometimes called internal resistance

to distinguish it from the resistance of the outer circuit, called external resistance. Resistance in general is denoted by R, electromotive force by E, and current by C.

A battery of R 2 ohms; sends a current of .035 ampere through a wire of R 48 ohms; what is the E. M. F. of the battery?

Solution: The resistance is $48 + 2 = 50$ ohms. By Rule 2 we have as the E. M. F. $50 \times .035 = 1.75$ volts.

A maximum difference of potential E. M. F. of 30 volts is maintained in a circuit and a current of 191 amperes is the result; what is the resistance of the circuit?

Solution: By Rule 3 the resistance is equal to $\frac{30}{191} = .157$ ohms.

In the same circuit several generators or galvanic couples may be included, some opposing the others, i. e. connected in opposition. All such can be conceived of as arranged in two sets, distributed according to the direction of current produced by the constituent elements, in other words, so as to put together all the generators of like polarity. The voltages of each set are to be added together to get the total E. M. F. of each set.

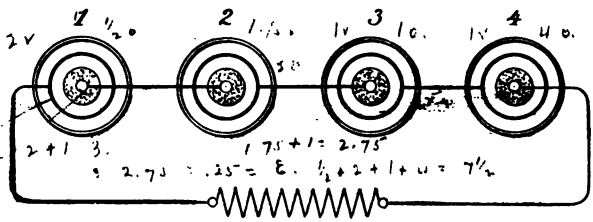
Rule 7. Where batteries or generators are in opposition, add together the E. M. F of all generators of like polarity, thus obtaining two opposed E. M. F.s. Subtract the smaller E. M. F. from the larger E. M. F. to

obtain the effective E. M. F. Then apply Ohm's law on this basis of E. M. F.

It will be understood that the resistances of all batteries or generators in series are added to give the internal resistance.

EXAMPLES.

There are four batteries in a circuit: Battery No. 1 of 2 volts, ½ ohm; Battery No. 2 of 1.75 volts, 2 ohms; Battery No. 3 of 1 volt, 1 ohm; Battery No. 4 of 1 volt, 4 ohms constants; Batteries 1 and 4 are in opposition to 2 and 3. What are the effective battery constants?



Solution: Voltage = (2 + 1) — (1.75 + 1) = .25 volt. Resistance = ½ + 2 + 1 + 4 = 7½ ohms, or .25 volt, 7½ ohms constants.

What current will such a combination produce in a circuit of 5 ohms resistance?

Solution: By Ohm's law, Rule 1, the current = .25 ÷ (7½ + 5) = .02 amperes.

A battery of 51 volts E. M. F. and 20 ohms resist-

ance has opposed to it in the same circuit a battery of 26 volts E. M. F. and 25 ohms resistance. A current of $\frac{1}{8}$ ampere is maintained in the circuit. What is the resistance of the wire leads and connections?

Solution: The effective E. M. F. is $51 - 26 = 25$ volts. By Rule 3 we have $25 \div \frac{1}{8} = 200$ ohms, as the total resistance. But the resistance of the batteries (internal resistance) is $20 + 25 = 45$ ohms. The resistance of leads, etc. (external resistance), is therefore $200 - 45 = 155$ ohms.

Portions of Circuits.

All portions of a circuit receive the same current, but the E. M. F., in this case termed preferably difference of potential, or drop or fall of potential, and the resistance may vary to any extent in different sections or fractions of the circuit. Ohm's Law applies to these cases also.

Examples.

An electric generator of unknown resistance maintains a difference of potential of 10 volts between its terminals connected as described. The terminals are connected to and the circuit is closed through a series of three coils, one of 100 ohms, one of 50 ohms, and one of 25 ohms resistance. The connections between these parts are of negligibly low re-

sistance. What difference of potential exists between the two terminals of each coil respectively?

Solution: The solution is most clearly reached by a statement of the proportion expressed in Rule 6, viz.: The electromotive force varies directly with the resistance. The resistance of the three coils is 175 ohms; calling them 1, 2, and 3, and their differences of potential E^1, E^2, and E^3, we have the continued proportion, 175 : 100 : 50 : 25 :: 10 volts : $E^1 : E^2 : E^3$. because by the conditions of the problem the total E. M. F. = 10. Solving the proportion by the regular rule, we find that $E^1 = 5.7$, $E^2 = 2.8$ and $E^3 = 1.4$ volts.

The same external circuit is connected to a battery of 30 ohms resistance. The difference of potential of the 100 ohm coil is found to be 30 volts. What is the difference of potential between the terminals of the battery, and what is the E. M. F. of the battery on open circuit, known as its voltage or E. M. F. (one of the battery constants)?

Solution: The total external resistance is 100 + 50 + 25 = 175 ohms. By Rule 6, we have 100 : 175 :: 30 volts : $x = 52\frac{1}{2}$ volts, difference of potential between the terminals of the battery. The current is found by dividing (Rule 1), the difference of potential of the 100 ohm coil by its resistance. This E. M. F. is 30. The current therefore is $\frac{30}{100}$ amperes. The total resistance of the circuit is that of the three coils or 175 ohms *plus* that of the battery or 30 ohms, a

total of 205 ohms. To maintain a current of $\frac{30}{100}$ amperes through 205 ohms (Rule 2), an E. M. F. is required equal to $\frac{30}{100} \times 205$ volts or $61\frac{1}{2}$ volts.

DIVIDED, BRANCHED OR SHUNT CIRCUITS.

A single conductor, from one terminal of a generator may be divided into one or more branches which may reunite before reaching the other terminal. Such branches may vary widely in resistance.

Rule 8. In divided circuits, each branch passes a portion of a current inversely proportional to its resistance.

EXAMPLES.

A portion of a circuit consists of two conductors, A and B, in parallel of A = 50, and B = 75 ohms, respectively; what will be the ratio of the currents passing through the circuit, which will go through each conductor?

Solution: The ratio will be current through A : current through B :: 75 : 50, which may be expressed fractionally, $\frac{1}{50} : \frac{1}{75}$.

Where more than two resistances are in parallel, the fractional method is most easily applied.

Three conductors of A = 25, B = 50, and C = 75 ohms are in parallel. What will be the ratio of currents passing through each one?

Solution: Fractionally A : B : C :: $\frac{1}{25} : \frac{1}{50} : \frac{1}{75}$.

Rule 9. To determine the amount of a given current that will pass through parallel circuits of different resistances, proceed as follows: Take the resistance of each branch for a denominator of a fraction having 1 for its numerator. In other words, for each branch write down the reciprocal of its resistance. Then reduce the fractions to a common denominator, and add together the numerators. Taking this sum of the numerators for a new common denominator, and the original single numerators as numerators, the new fractions will express the proportional currents as fractions of one. If the total amperage is given, it is to be multiplied by the fractions to give the amperes passed by each branch. The solution can also be done in decimals.

EXAMPLES.

A lead of wire divides into three branches; No. 1 has a resistance of 10,000 ohms, No. 2 of 39 ohms, and No. 3 of $\frac{1}{3}$ ohm. They unite at one point. What proportion of a unitary current will pass each branch?

Solution: The proportion of currents passed are as $\frac{1}{10000}$: $\frac{1}{39}$: $\frac{1}{\frac{1}{3}}$ or 3. Reducing to a common denominator, these become $\frac{39}{390000}$: $\frac{10000}{390000}$: $\frac{1170000}{390000}$. The proportions of the numerators is the one sought for; taking the sum of the numerators as a common denominator, we have in common fractions the following proportions of any current passed by the three branches. No. 1, $\frac{39}{1180039}$; No. 2, $\frac{10000}{1180039}$; No. 3, $\frac{1170000}{1180039}$.

Four parallel members of a circuit have resistances respectively of 25, 85, 90, and 175 ohms; express

decimally the ratio of a unitary current that will pass through them.

Solution: The ratio is as $\frac{1}{25} : \frac{1}{85} : \frac{1}{90} : \frac{1}{175}$, or reducing to decimals (best by logarithms), .04 : .011765 : .011111 : .0057. Adding these together, we have .068576, which must be multiplied by 14.582 to produce unity. Multiplying each decimal by 14.58 (best by logarithms), we get the unitary ratio as .5832 : .17153 : .1620 : .08310, whose sum is 1.0000.

Unless logarithms are used, it is far better to work by vulgar fractions.

A current of .71 amperes passes through two branches of a circuit. One is a lamp with its connections of 115 ohms resistance; another is a resistance coil of 275 ohms resistance. What current passes through each branch?

Solution: The proportions of the current are as $\frac{1}{115} : \frac{1}{275}$ or reduced to a common denominator and to their lowest terms $\frac{55}{6325} : \frac{23}{6325}$. Proceeding as before, and taking the sum of the numerators $(55 + 23 = 78)$, as a common denominator, we find that the lamp passes $\frac{55}{78}$, and the resistance coil $\frac{23}{78}$ of the whole current. Multiplying the whole current, .71 by $\frac{55}{78}$, we get $\frac{3905}{7800}$ amperes, or $\frac{1}{2}$ ampere for the lamp, leaving .21 or a little over $\frac{1}{5}$ ampere for the resistance coil.

Another problem in connection with parallel branches of a circuit is the combined resistance of parallel circuits. This is not a case of summa-

tion, for it is evident that the more parallel paths there are provided for the current, the less will be the resistance.

Rule 10. In shunt circuits, the resistance of the combined shunts is expressed by the reciprocal of the sum of the reciprocals of the resistances.

EXAMPLE.

Two leads of a 50 volt circuit (leads differing in potential by 50 volts), are connected by a 20 ohm motor. A 50 ohm lamp and 1000 ohm resistance coil are connected in parallel or shunt circuit therewith, what is the combined resistance? and the total current?

Solution: The reciprocal of resistance is conductance, sometimes expressed as mhos. (Rule 19.) The conductance of the three shunts is equal to $\frac{1}{20} + \frac{1}{50} + \frac{1}{1000}$ mhos $= \frac{50}{1000} + \frac{20}{1000} + \frac{1}{1000} = \frac{71}{1000}$ mhos. The reciprocal of conductance is resistance. The combined resistance is therefore $\frac{1000}{71}$ ohms $=$ 14.09 ohms. The current is $\frac{50}{14.09}$ or 3.5 amperes.

Rule 11. The combined resistance of two parallel circuits is found by multiplying the resistances together, and dividing the product by the sum of the resistances. Where there are several circuits, any two can be treated thus, and the result combined in the same way with another circuit, and so on to get the final resistance.

$$R = \frac{r \times r^1}{r + r^1}$$

EXAMPLE.

Four conductors in parallel have resistances of 100 — 50 — 27 — 19 ohms. What is their combined resistance?

Solution: Combining the first and second, we have $\frac{100 \times 50}{100 + 50} = 33\frac{1}{3}$ ohms. Combining this with the resistance of the third wire, we have $\frac{33\frac{1}{3} \times 27}{33\frac{1}{3} + 27} = 14.9$ ohms. Combining this with the resistance of the fourth wire, we have $\frac{14.9 \times 19}{14.9 + 19} = 8.3$ ohms. The result is, of course, identical by whatever rule obtained.

Rule 12. When all the parallel circuits are of uniform resistance, as in multiple arc incandescent lighting, the resistance of the combined circuits is found by dividing the resistance of one circuit by the number of circuits.

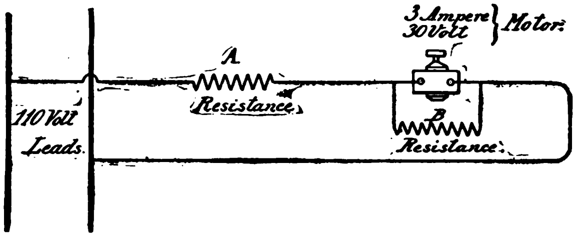
$$R = \frac{r}{n}$$

EXAMPLES.

There are fifty lamps of 100 ohms resistance each in multiple arc connection. What is their combined resistance?

Solution: $\frac{100}{50} = 2$ ohms.

A motor can take 3 amperes of currents at 30 volts safely without burning out or heating injuriously. A 110 volt incandescent circuit is at hand. The motor is to be connected across the leads so as to receive the above amperage. A shunt or branch of some resistance is carried around it, and a resistance coil intervenes between the united branches and one of the main leads. The resistance of the coil is 20 ohms. What should the resistance of the shunt be?



Solution: The resistance of the motor (Ohm's Law, Rule 3), is found by dividing the E. M. F. by the resistance—$30 \div 3 = 10$ ohms. By Rule 5 the resistance of the coil in series (20 ohms) must be to the combined (not added) resistance of the motor and shunt coil, as $110 - 30$ (total voltage minus voltage for motor) : 30 (voltage for motor) or $20 : x :: 80 : 30 \therefore x = 7.5$ combined resistance of parallel or shunt coil and motor. The reciprocal of 7.5 (conductance, Rule 19), may be expressed as $\frac{100}{750}$ths of the combined (in this case added) conductances of shunt coil and motor. The conductance of the motor is equal to the reciprocal of 10 which may be expressed as $\frac{1}{10}$ or as $\frac{75}{750}$. The conductance of the shunt coil must therefore be $\frac{100}{750} - \frac{75}{750} = \frac{25}{750} = \frac{1}{30}$ mho. The reciprocal of this gives the resistance of the shunt coil which is 30 ohms. The total current going through the system by Ohm's law is $\frac{110}{7\frac{1}{2} + 20} = 4$ amperes. The resistance of the shunt coil—30 ohms—is to that of the motor in parallel with it—10 ohms—as the current received by the motor is to

that received by the coil, a ratio of 30 : 10 or 3 : 1 giving 3 amperes for the motor and 1 ampere for the coil. This is a proof of the correctness of operations.

Two conductors through which a current is passing are in parallel circuit with each other. One has a resistance of 600 ohms. The other has a resistance of 3 ohms. A wire is carried across from an intermediate point of one to a corresponding point of the other. It is attached at such a point of the first wire that there are 400 ohms resistance before it and 200 after it. Where must it be connected to the other in order that no current may pass?

Solution: The E. M. F. up to the point of connection of the bridge or cross wire is to the total E. M. F. in the 600 ohm wire as 400: 600 or as 2: 3. The other wire which by the conditions has the same drop of potential in its full length must be divided therefore in this ratio. The bridge wire must therefore connect at 2 ohms from its beginning, leaving 1 ohm to follow. The principle here illustrated can be proved generally and is the Wheatstone Bridge principle.

CHAPTER III.

RESISTANCE AND CONDUCTANCE.

RESISTANCE OF DIFFERENT CONDUCTORS OF THE SAME MATERIAL.

Conductors are generally circular in section. Hence they vary in section with the square of their diameters. The rule for the resistance of conductors is as follows:

Rule 13. The resistance of conductors of identical material varies inversely as their section, or if of circular section inversely as the squares of their diameters, and directly as their lengths.

EXAMPLE.

1. A wire *a*, is 30 mils in diameter and 320 feet long; another *b*, is 28 mils in diameter and 315 feet long. What are their relative resistances?

Solution: Calling the resistances $R^a : R^b$ we would have the inverse proportion if they were of equal lengths $R^b : R^a :: 30^2 : 28^2$ or as $900 : 784$. Were they of equal diameter the direct proportion would hold for their lengths: $R^b : R^a :: 315 : 320$. Combining the two by multiplication we have the compound proportion $R^b : R^a :: 900 \times 315 : 784 \times 320$ or as $283,500 : 250,880$, or as $28 : 25$ nearly. The

combined proportions could have been originally expressed as a compound proportion thus: $R^b : R^a :: 30^2 \times 315 : 28^2 \times 320$.

For wires of equal resistance the following is given.

Rule 14. The length of one wire multiplied by the square of the diameter of the other wire must equal the square of its own diameter multiplied by the length of the other if their resistances are equal. Or multiply the length of the first wire by the square of the diameter of the second. This divided by the length of the second will give the square of the diameter of the first wire; or divided by the square of the diameter of the first will give the length of the second. $ld'^2 = l'd^2$

EXAMPLES.

1. There are three wires, *a* is 2 mils, *b* is 3 mils, and *c* is 4 mils in diameter; what length must *b* and *c* have to be equal in resistance to ten feet of *a*?

Solution: Take *a* and *c* first and apply the rule, $10 \times 4^2 \div 2^2 = 40$ feet; then take *a* and *b* $10 \times 3^2 \div 2^2 = 22\frac{1}{2}$ feet. To prove it compare *a* and *c* directly by the same rule $22\frac{1}{2} \times 4^2 \div 3^2 = 40$. As this gives the same result as the first operation, we may regard it as proved.

A conductor is 75 mils in diameter and 79 feet long; how thick must a wire 1264 feet long be to equal it in resistance?

Solution: $75^2 \times 1264 \div 79 = 7,110,000 \div 79 = 90,000$. The square root of this amount is 300 which is the required diameter.

For problems involving the comparison of wires of unequal resistance the rule may be thus stated:

Rule 15. Multiply the square of the diameter of each wire by the length of the other. Of the two products divide the one by the other to get the ratio of resistance of the dividend to that of the divisor taken at unity. The term including the length of a given wire is the one expressing the relative resistance of such wire.

EXAMPLES.

A wire is 40 mils in diameter, 3 miles long and 40 ohms resistance. A second wire is 50 mils in diameter and 9 miles long. What is its resistance?

Solution: $9 \times 40^2 = 14{,}400$ relative resistance of the first wire. $3 \times 50^2 = 7{,}500$ relative resistance of second wire. $14{,}400 \div 7{,}500 = 1.92$ — ratio of resistance of second wire to that of first taken at unity. But the latter resistance really is 40 ohms. Therefore the resistance of the second wire is $40 \times 1.92 = 76.80$ ohms.

The result may also be worked out thus:

$40^2 \times 9 = 14{,}400 =$ relative resistance of the 3 mile wire.

$50^2 \times 3 = 7500 =$ relative resistance of the 9 mile wire.

$14{,}400 \div 7500 = 1.92 =$ ratio of 9 mile (dividend) to 3 mile (divisor) wire.

$\therefore$ 40 ohms $\times$ 1.92 = 76.8 ohms.

A length of a thousand feet of wire 95 mils in diameter has 1.15 ohms resistance; what is the di-

ameter of a wire of the same material of which the resistance of 1000 feet is 10.09 ohms? (R. E. Day, M. A.).

Solution: 10.09 ÷ 1.15 = 8.77 ratio of resistances. If we divide 1000 by 8.77 we obtain a length of the first wire which reduces the question to one of identical resistances. 1000 ÷ 877 = 114 feet. Then applying Rule 14, $114 \times 95^2 \div 1000 = 1037.88$. This is the square of the diameter of the other wire. Its square root gives the answer: 32.2 mils.

Specific Resistance.

Specific resistance is the resistance of a cube of one centimeter diameter of the substance in question between opposite sides. It is expressed in ohms for solutions and in microhms for metals. From it may be determined the resistance of all volumes, generally prisms or cylinders, of substance. Very full tables of Specific Resistance are given in their place.

Rule 16. The resistance of any prism or cylinder of a substance is equal to its specific resistance multiplied by its length in centimeters and divided by its cross-sectional area in square centimeters. If the dimensions are given in inches or other units of measurements they must be reduced to centimeters by the table.

$$R = \frac{Sp.\ R \times l}{a}$$

Examples.

An electro-plater has a bath of sulphate of copper, sp. resistance 40 ohms. His electrodes are each 1

foot square and 1 foot apart. What is the resistance of such a bath?

Solution: By the table 1 square foot = 929 sq. cent. and 1 foot = 30.4797 cent. $\therefore$ Resistance = $40 \times 30.4797 \div 929 = 1.31$ ohms.

Where the electrodes in a solution are of uneven size take their average size per area. The facing areas are usually the only ones calculated, as owing to polarization the rear faces are of slight efficiency, and where the electrodes are nearly as wide as the bath or cell the active prism is practically of cross-sectional area equal to the area of one side of a plate.

In a Bunsen battery the specific resistances of the solutions in inner and outer cells were made alike, each equalling 9 ohms. The central element was a ½ inch cylinder of electric light carbon. The outer element was a plate of zinc 6 inches long bent into a circle. When there were 2 inches of solution in the cell what was the resistance?

Solution: Area of carbon = $\frac{\pi}{2} \times 2 = 3.14$ square inches. Area of zinc = $2 \times 6 = 12$ square inches. This gives an average facing area of $(12 + 3.14) \div 2 = 7.57$ square inches = 48.38 sq. cent. The distance apart = ¾ inches (nearly) = 1.9049 cent. $\therefore$ Resistance = $9 \times 1.9049 \div 48.38 = .354$ ohms.

For wires, the specific resistance of metals being given in microhms, the calculation may be made in microhms, or in ohms directly. As wire is cylindri-

cal a special calculation may be made in its case to reduce area of cross section to diameter. This may readily be taken from the table of wire factors, thus avoiding all calculation.

Rule 17. The resistance in microhms of a wire of given diameter in centimeters is equal to the product of the specific resistance by 1.2737 by the length in centimeters divided by the square of the diameter in centimeters.

$$R = \frac{\text{Sp. Res.} \times 1.2737 \times l}{d^2}$$

EXAMPLES.

The Sp. Res. of copper being taken at 1.652 microhms what is the resistance of a meter and a half of copper wire 1 millimeter thick?

Solution: The diameter of the wire (1 millimeter) is .1 centimeter. The square of .1 is .01. The length of the wire ($1\frac{1}{2}$ meter) is 150 centimeters. Its resistance therefore is $1.652 \times 1.2737 \times 150 \div .01 = 31{,}561$ microhms or .031,561 ohms.

UNIVERSAL RULE FOR RESISTANCES.

Into the problem of resistances of one or two wires eight factors can enter, these are the lengths, sectional areas, specific resistances and absolute resistances of two wires. Their relation may be expressed by an algebraic equation, which by transposition may be made to fit any case. The rule is arithmetically expressed by adopting the method of cancellation, drawing a vertical line and placing on

the left side, factors to be multiplied together for a divisor, and on the right side factors to be multiplied together for a dividend. In the expression of the rule as below the quotient is 1, in other words the product of all the factors on the left hand of the line is equal to that of all the factors on the right hand. Calling one wire *a* and the other *b* we have the following expression:

Resistance of *b*	Resistance of *a*
Specific Resistance of *a*	Specific Resistance of *b*
Length of *a*	Length of *b*
Cross-sectional area of *b*	Cross-sectional area of *a*

Rule 18. Substitute in the above expression the values of any factors given. Substitute for factors not given or required the figure 1 or unity. Such a value determined by division must be given to the required factor and substituted in its place as will make the product of the left-hand factors equal to that of the right-hand factors. Only one factor can be determined, and all factors not given are assumed to be respectively equal for both conductors.

EXAMPLES.

If the resistance of 500 feet of a certain wire is .09 ohms what is the resistance of 1050 feet of the same wire?

Solution: The cross sectional areas and specific resistance not being given are taken as equal. (This of course follows from the identical wire being referred to.) The vertical line is drawn and the values substituted :

Resistances:	Resistance of required wire	.09
Lengths:	500	1050

(Other factors omitted as unnecessary.)
$1050 \times .09 \div 500 = .189$ ohms.

What is the diameter of a wire 2 miles long of 23 ohms resistance, if a mile of wire of similar material of seventy mils diameter has a resistance of 10.82 ohms?

Solution. We use for simplicity the square of the diameter in place of the cross sectional area of the known wire, thus:

Resistances:	23	10.82
Lengths:	1	2
Areas:	Unknown	70^2

As the specific resistances are identical they are not given.
$2 \times 70^2 \times 10.82 \div 23 \times 1 = 4610$ square of diameter required: $4610^{\frac{1}{2}} = 68$ mils.

What must be the length of an iron wire of cross-sectional area 4 square millimeters to have the same resistance as a wire of pure copper 1000 yards long, of cross-sectional area 1 square millimeter, taking the conductance of iron as $\frac{1}{7}$ that of copper? (Day).

Solution:

Specific Resistances:	1	7 (i.e. the reciprocal of conductance)
Lengths:	1000	Unknown
Cross-sectional areas:	4	1

As the resistances are identical they are not given.

Solving we have $1000 \times 4 \div 7 = 571\frac{3}{7}$ yards.

There are two conductors, one of 35 ohms resistance, 1728 feet long and 12 square millimetres cross-sectional area and specific resistance 7: the other of 14 ohms resistance, 432 feet long and 8 square millimetres cross-sectional area. What is its specific resistance?

Resistances:	35	14
Specific Resistances Unknown		7
Lengths:	432	1728
Cross-Sectional areas:	12	8

By cancellation this reduces to $14 \times 8 \div 5 \times 3 = 7.4$ Specific Resistance.

In these cases it is well to call one wire *a* and the other *b*, and to arrange the given factors in two columns headed by these designations. Then the formula can be applied with less chance of error. Thus for the last two problems the columns should be thus arranged.

	a	*b*
Area,	4 sq. mil.	1
L.	Unknown	1000 yards
Sp. Res.	7	1

	a	*b*
Resist.	35 ohms	14 ohms.
L.	1728 feet	432 feet
Area,	12 sq. mils	8 sq mils.
Sp. Res.	7	required.

From such statements of known data the formula can be conveniently filled up.

RESISTANCE OF WIRES REFERRED TO WEIGHT.

The weight of equal lengths of wire is in proportion to their sections. The problems involving weight therefore can be reduced to the Rules already given.

Problem. A wire, A, is 334 feet long and weighs 25 oz.; another, B, is 20 feet long and weighs 1 oz. what are the relative resistances?

Solution: 20 feet of the wire "A" weigh $\frac{20}{334} \times 25 = 1.50$ oz. The weights of equal lengths of A and B respectively are as 1.50 : 1.00 which is also the inverse ratio of the resistances of equal lengths. By compound proportion Rule we have R. of "A" : R. of "B" :: 1 × 334 :: 1.50 × 20; reducing to 16.7 : 1.5 or 11.1 : 1.0 or the wire "A" has about eleven times the resistance of the wire "B."

Solution: By general Rule for resistance (Rule 18). Taking 1.50 : 1.00 as the ratio of cross-sectional areas and taking the resistance of the long wire A as 1 we have:

Resistances:	1	Unknown
Lengths:	20	334
Cross-sectional area:	1.50	1

Resistance of B $= 1.50 \times 20 \div 334 = .0899$ or about $\frac{1}{11}$ as before.

Conductance.

Conductance is the reciprocal of resistance and is sometimes expressed in units called MHOS, which is derived from the word ohm written backwards.

Rule 19. To reduce resistance in ohms to conductance in mhos express its reciprocal and the reverse.

$$K = \frac{1}{R}$$

Examples.

A wire has a resistance of $\frac{18}{126}$ ohms, what is its conductance?

Solution: $126 \div 18 = 7$ mhos.

Reduce a conductance of $1\frac{19}{25}$ to ohms.

Solution: $1\frac{19}{25} = \frac{44}{25}$ mhos which gives $\frac{25}{44}$ ohms.

It is evident that the data for problems or that constants could be given in mhos instead of ohms. In some ways it is to be regretted that the positive quality of conductance was not adopted at the outset instead of the negative quality of resistance. One or two illustrations may be given in the form of examples involving conductance.

Express Ohm's law in its three first forms in conductance.

Solution: This is done by replacing the factor "resistance" by its reciprocal. Thus, Rule 1 reads for conductance: "The current is equal to the electromotive force multiplied by the conductance" ($C = EK$)—Rule 2 as "The electromotive force is equal to the current divided by the conductance" ($E = \frac{C}{K}$)—Rule 3 as "The conductance is equal to the current divided by the electromotive force." ($K = \frac{C}{E}$)

A circuit has a resistance of .5 ohm and an E. M. F. of 50 volts; determine the current, using conductance method.

Solution: The conductance $= \frac{1}{.5} = 2$ mhos. The current $= 50 \times 2 = 100$ amperes.

In a circuit a current of 20 amperes is maintained through $2\frac{3}{5}$ ohms. Determine the E. M. F. using conductance.

Solution: The conductance $= \frac{1}{2\frac{3}{5}} = \frac{10}{26}$ mhos. E. M. F $= 20 \div \frac{10}{26} = 52$ volts.

Assume a current of 30 amperes and an E. M. F. of 50 volts, what is the conductance and resistance?

Solution: Conductance $= 30 \div 50 = .6$ mho. Resistance $= 1 \div .6 = 1.667$.

CHAPTER IV.

POTENTIAL DIFFERENCE.

Drop of Potential in Leads and Size of Same for Multiple Arc Connections.

Subsidiary leads are leads taken from large sized mains of constant E. M. F. or from terminals of constant E. M. F. to supply one or more lamps, motors, or other appliances. A constant voltage is maintained in the mains or terminals. There is a drop of potential in the leads so that the appliances always have to work at a diminished E. M. F. The E. M. F. of the leads is known, the requisite E. M. F. and resistance of the appliance is known, a rule is required to calculate the size of the wire to secure the proper results. It is based on the principle that the drop or fall in potential in portions of integral circuits varies with the resistance. (See Ohm's law). A rule is required for a single appliance or for several connected in parallel.

Rule 20. The resistance of the leads supplying any appliance or appliances for a desired drop in potential within the leads is equal to the reciprocal of the current of the appliances multiplied by the desired drop in volts.

EXAMPLE.

A lamp, 100 volts × 200 ohms, is placed 100 feet from the mains, in which mains a constant E. M. F. of 110 volts is maintained. What must be the resistance of the line per foot of its length; and what size copper wire must be used?

Solution: The lamp current is obtained (Ohm's law) by dividing its voltage by its resistance, ($\frac{100}{200} = \frac{1}{2}$ ampere). The reciprocal of the current is $\frac{2}{1}$; multiplied by the drop ($\frac{2}{1} \times 10 = 20$) it gives the resistance of the line as 20 ohms. As the lamp is 100 feet from the mains there are 200 feet of the wire. Its resistance per foot is therefore $\frac{20}{200} = \frac{1}{10}$ ohm or it is No. 30 A. W. G. (about).

For several appliances in parallel on two leads a similar rule may be applied. There is inevitably a variation in E. M. F. supplied to the different appliances unless resistances are intercalated between the appliances and the leads.

Rule 21. The E. M. F. of the main leads or terminals the factors of the lamps or other appliances, their number and the distance of their point of connection are given. The combined resistance is found by Rules 8 to 12. Then by Rule 20 the resistance of the leads is calculated.

EXAMPLE.

A pair of house leads includes 260 feet of wire, or 130 in each lead. Six 50 volt 100 ohm lamps are connected thereto at the ends. The drop is to be 5

volts, giving 55 volts in the main leads. Required the total resistance of and size of wire for the house leads.

Solution: The resistance of six 100 ohm lamps in parallel is $100 \div 6 = 16.66$ ohms. The current required is by Ohm's law $50 \div 16.66$ or 3 amperes. Its reciprocal multiplied by the drop, ($\frac{1}{3} \times 5 = \frac{5}{3} = 1\frac{2}{3}$ ohms) gives the required resistance $= 1\frac{2}{3}$ ohms. This, divided by 260 feet gives the resistance per foot as .0064 ohm, corresponding by the table to No. 18 A. W. G.

A rule for the above cases is sometimes expressed otherwise, being based on the proportion: Resistance of appliances is to resistance of leads as 100 *minus* the drop expressed as a percentage is to the drop expressed as a percentage. This gives the following:

Rule 22. The resistance of the leads is equal to the combined resistance of the appliances multiplied by the percentage of drop and divided by 100 *minus* the percentage of drop.

Problem. Take the data of last problem and solve.

Solution: The percentage of drop is $\frac{5}{55} = 9\%$. The resistance of the leads $= \frac{16.66 \times 9}{100 - 9} = \frac{149.94}{91} = 1\frac{2}{3}$ ohms about.

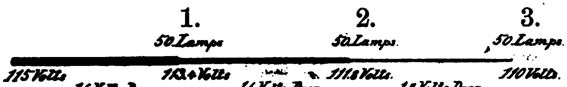
Note.—To obtain accurate results the figures of percentage, etc., must be carried out to two or more decimal places. Rules 20 and 21 are to be preferred to any percentage rule. Also see Rule 23.

Where groups of lamps are to be connected along a pair of leads but at considerable intervals, the succeeding sections of leads have to be of diminishing size. The same problem arises in calculating the sizes of street leads. The identical rule is applied, care being taken to express correctly the exact current going through each section of the lead. The calculation is begun at the outer end of the leads. A diagram is very convenient; it may be conventional as shown below.

EXAMPLES.

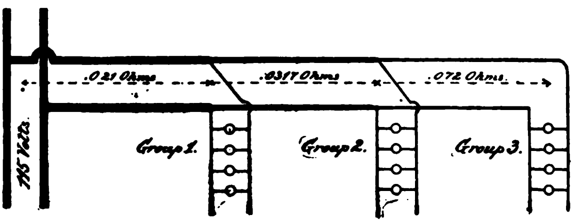
At three points on a pair of mains three groups of fifty 220 ohm lamps in parallel are connected; a total drop of 5 volts is to be divided among the three groups, thus: 1.6 volts — 1.6 volts — 1.8 volts. The initial E. M. F. is 115 volts; what must be the resistances of the three sections of wire?

Solution: The following diagram gives the data as detailed above:



Starting at group 3 we have 50 lamps in parallel each of 220 ohms resistance, giving a combined resistance (Rule 12) of 4.4 ohms and a total current (Ohm's law) of 110 $\div$ 4.4 = 25 amperes. The resistance of section 2—3 is by the present rule $\frac{1}{25}$ $\times$

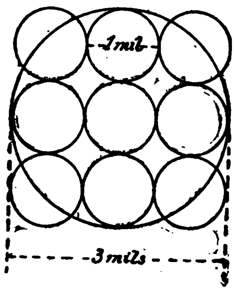
1.8 = .072 ohms. Taking group 2 the current through this group of lamps is 111.8 ÷ 4.4 = 25.41 amperes. The section 1—2 has to pass also the current 25 amperes for group 3 giving a total current of 25 + 25.41 = 50.41 amperes. The resistance of section 1—2 is therefore $\frac{1}{50.41} \times 1.6 = .0317$ ohm. Taking group 1 the current for its lamps is 113.4 ÷ 4.4 = 25.7 amperes. The total current through section 0—1 is therefore 25 + 25.4 + 25.7 = 76.1 amperes. The resistance of the section is $\frac{1}{76.0} \times 1.6$ = .021 ohms. Arranging all these data upon a diagram we have the full presentation of the calculation in brief as below:



CHAPTER V.

CIRCULAR MILS.

A MIL is $\frac{1}{1000}$ of an inch. The area of a circle, one mil in diameter, is termed a circular mil. The area of the cross-section of wires is often expressed in circular mils. Thus a wire, 3 mils in diameter, has an area of 9 circular mils, as shown in the cut. A



circular mil is .7854 square mil. Rules for the sizes of wires for given resistances are often based on circular mils, and include a constant for the conductivity of copper. By the table of specific resistances, the values found can be reduced to wires of iron or other metals.

A commercial copper wire, one foot long, and one circular mil in section, has a resistance of 10.79 ohms at 75° F. This is, of course, largely an assumption, but is taken as representing a good average. No two samples of wire are exactly alike, and many vary largely. From Rule 13, and from the above constant, we derive the following rules:

Rule 23. The resistance of a commercial copper wire is equal to its length divided by the cross-section in circular mils, and multiplied by 10.79.

EXAMPLE.

A wire is 1050 feet long, and has a cross-section of 8234 circular mils. What is its resistance?

Solution: $1050 \times 10.79 \div 8234 = 1.37$ ohms.

Rule 24. The cross-section of a wire in circular mils is equal to its length divided by its resistance, and multiplied by 10.79.

EXAMPLE.

A wire is 1050 feet long, and has a resistance of .68795 ohms. What is its cross-section in circular mils?

Solution: $1050 \times 10.79 \div .68795 = 16{,}468$ circular mils.

Rule 25. The cross-section of the wires of a pair of leads in circular mils for a given drop expressed in percentage is equal to the product of the length of leads by the number of lamps (in parallel), by 21.58, by the difference between 100 and the drop, the whole divided by the resistance of a single lamp multiplied by the drop.

$$A = \frac{ln \times 21.58 \times (100 - e)}{er}$$

EXAMPLE.

Fifty lamps are to be placed at the end of a double lead 150 feet long (= 300 feet of wire). The resistance of a lamp is 220 ohms. What size must the wire be for 5% drop?

Solution: $150 \times 50 \times 21.58 \times (100 - 5) \div (220 \times 5) = 13{,}977.9$ circular mils.

In these calculations and in the calculations given on page 48 it is important to bear in mind that the percentage is based upon the difference of potential at the beginning of the leads or portion thereof under consideration; in other words upon the highest difference of potential within the system or the portion of the system treated in the calculation.

CHAPTER VI.

SPECIAL SYSTEMS.

THREE WIRE SYSTEM.

As there are three wires in the three wire system, where there are two in the ordinary system, and as each of the three wires is one quarter the size of each of the two ordinary system wires, the copper used in the three wire system is three-eighths of that used in the ordinary system.

In the three wire system the lamps are arranged in sets of two in series. Hence but one-half the current is required. The outer wires, it will be noticed, have double the potential of the lamps. Hence to carry one-half the current with double the E. M. F., a wire of one quarter the size used in the ordinary system suffices.

Rule 26. The calculations for plain multiple arc work apply to the three wire system, as regards size of each of the three leads, if divided by 4.

While the central or neutral wire will have nothing to do when an even number of lamps are burning on each side, and may never be worked to its full capacity, there is always a chance of its having to carry a full current to supply half the lamps (all on

one side). Hence it is made equal in size to the others.

Alternating Current System.

The rules already given apply in practice to this system also, although theoretically Ohm's law and those deduced from it are not correct for this current. A calculation has to be made to allow for the conversion from primary to secondary current in the converter as below.

Note.—The ratio of primary E. M. F. to secondary is expressed by dividing the primary by the secondary, and is termed *ratio of conversion.* Thus in a 1000 volt system with 50 volt lamps in parallel the *ratio of conversion* is $1000 \div 50 = 20$.

Rule 27. The current in the primary is equal to the current in the secondary divided by the ratio of conversion.

Examples.

On an alternating current circuit whose ratio of conversion = 20, there are 1000 lamps, each 50 volt; 50 ohms. When all are lighted what is the primary current?

Solution: By Ohm's law the secondary current is 1000×1 (each lamp using $\frac{50}{50} = 1$ ampere, Ohm's law) = 1000 amperes. $1000 \div 20 = 50$ amperes is the primary current.

The current being thus determined the ordinary rules all apply exactly as given for direct current work.

Given 650 lamps, 50 volt 50 ampere each, 3600 feet from the station. The primary circuit pressure is 1031 volts. A drop of 3% is to be allowed for in the primary leads. What is the resistance of the primary wire?

Solution: Current of a single lamp = 50 ÷ 50 = 1 ampere; current of 650 lamps = 650 amperes, current of primary 650 ÷ 20 = 32½ amperes, drop of primary = 1031 × 3% = 30.9 volts, resistance of primary (Rule 20) $\frac{1}{32\frac{1}{2}}$ × 30.9 = .9516 ohm.

Rule 28. For obtaining the size of the primary wire in circular mils, calculate by Rule 25, and divide the result by the square of the ratio of conversion.

EXAMPLE.

Take data of last problem and solve.

Solution: [3600 × 650 × 21.58 × (100—3) ÷ (50 × 3)] ÷ 20^2 = 81,637 circular mils.

The two last examples may be made to prove each other, thus:

The total length of leads is 3600 × 2 = 7200 feet. If of 1 mil thickness its resistance would be 7200 × 10.79 = 77,688 ohms. As resistance varies inversely as the cross sectional area we have the proportion

.9516 : 77,688 :: 1 : x which gives x = 81,639 circular mils corresponding within limits to the result obtained by Rule 28.

In all cases of this sort where percentage is expressed the statement in the last paragraph on page

45 should be kept in mind. The ratio of conversion must be based on the E. M. F. at the coil (in this case on $1031 - 31 = 1000$ volts) not on the E. M. F. at the beginning of the leads or portion thereof considered in the calculation. The percentage of drop must be subtracted before the ratio of conversion is calculated.

For winding the converters, the following is the rule :

Rule 29. The convolutions of the primary are equal in number to the product of the convolutions of the secondary multiplied by the ratio of conversion, and vice versa.

EXAMPLES.

The ratio of conversion of a coil is 20; there are 1000 convolutions of the secondary. How many should there be of the primary?

Solution: $1000 \times 20 = 20{,}000$ convolutions.

There are in a coil 5000 convolutions of the primary; its ratio of conversion is 50. How many convolutions should the secondary have?

Solution: $5000 \div 50 = 100$ convolutions.

CHAPTER VII.

WORK AND ENERGY.

ENERGY AND HEATING EFFECT OF THE CURRENT.

IT has been shown experimentally by Joule that the total quantity of heat developed in a circuit is equal to the square of the current multiplied by the resistance. This is equal, by Ohm's law, to the square of the E. M. F. divided by the resistance, which again reduces to the E. M. F. multiplied by the current. Each of these expressions has its own application, and they may be given as three rules.

Rule 30. The energy or heat developed in circuits is in proportion to the square of the current multiplied by the resistance.

$$H = C^2 R.$$

EXAMPLES.

An electric lamp has a resistance of 50 ohms; it is connected to a street main by leads of 2½ ohms resistance. What proportion of heat is wasted in the house circuit?

Solution: The current being the same for all parts of the circuit, the heat developed is in proportion to the resistance, or as 2½ : 50 equal to 1 : 20. The

heat developed in the wire is wasted, therefore the waste is $\frac{1}{10}$ of the total heat developed.

The internal resistance of a battery is equal to that of 3 meters of a particular wire. Compare the quantities of heat produced both inside and outside the battery when its poles are connected with one meter of this wire with the quantities produced in the same time when they are connected by 37 meters of the same wire. (*Day.*)

Solution: The relative currents produced in the two cases are found (Ohm's law) by dividing the E. M. F. of the battery (a constant quantity = E) by the relative resistance. As the battery counts for the resistance of 3 meters of wire, the relative resistances are as 4 : 40. Were the same current developed in both cases these figures would give the desired ratio. But as the current varies it has to be taken into account. To determine the relative battery heat only, we may neglect resistance of the battery, as it is a constant for both cases, the battery remaining identical. By Ohm's law the currents are in the ratio of $\frac{E}{4} : \frac{E}{40}$ and their squares are in proportion to $\frac{E^2}{16} : \frac{E^2}{1600} = 100 : 1$. By the rule this is the proportion of the heats developed in the battery alone, with the short wire 100, with the long wire 1. For the heating effects on the outside circuit, as resistance and current both vary, the full formula of the rule must be applied. The ratio of the heat in the short wire connection to that in the long

wire connection is as $(\frac{E}{4})^2 \times 1 : (\frac{E}{40})^2 \times 37 = 100 \times 1 : 37 \times 1$. The ratio therefore is as 100 is to 37 for the total heat produced in the circuit which includes battery and connections.

Owing to irregular working of a dynamo, an incandescent lamp receives sometimes the full amount or $\frac{1}{2}$ ampere of current; at other times as little as $\frac{10}{45}$ ampere. What proportion of heat is developed in it in both cases, assuming its resistance to remain sensibly the same?

Solution: By the rule the ratio is $(\frac{1}{2})^2 : (\frac{10}{45})^2$ or $\frac{1}{4} : \frac{100}{2025} = 2025 : 400$. The diminution of current therefore cuts down the heat to $\frac{1}{5}$ the proper amount.

Rule 31. The energy or heat developed in a circuit is in proportion to the square of the electromotive force divided by the resistance.

$$H = \frac{E^2}{R}$$

EXAMPLES.

There are two Grove batteries, each developing 1.98 volts E. M. F. One has an internal resistance of $\frac{1}{10}$ ohm; the other of $\frac{1}{4}$ ohm. They are placed in succession on a circuit of 2 ohms resistance. What is the ratio of heats developed by the batteries in each case?

Solution: As the E. M. F. is constant it may be taken as unity. Then for the two cases we have $\frac{1}{2\frac{1}{10}} : \frac{1}{2\frac{1}{4}} = 2\frac{1}{4} : 2\frac{1}{10}$ as the ratio of heat produced.

A battery of one ohm resistance and two volts E.

M. F. is put in circuit with 4 ohms resistance. Another battery of 4 ohms and 1 volt is connected through 1 ohm resistance. What ratio of heat is developed in each case?

Solution: $\frac{2 \times 2}{5} : \frac{1 \times 1}{5}$ or 4 : 1.

Rule 32. The energy or heat developed in a circuit is in proportion to the E. M. F. multiplied by the current.

$$H = EC$$

EXAMPLES.

Take data of last problem and solve.

Solution: For first battery, by Ohm's law, current $= \frac{2}{5}$ ampere; for the second, current $= \frac{1}{5}$ ampere. The heat developed, is by the present rule, in the proportion as $\frac{2}{5} \times 2 : \frac{1}{5} \times 1$ or 4 : 1.

Compare the heat developed in a 100 volt 200 ohm lamp and in a 35 volt 35 ohm lamp and in a 50 volt 50 ohm lamp.

Solution: The currents (Ohm's law) are: $\frac{100}{200}$, $\frac{35}{35}$ and $\frac{50}{50}$ in amperes $= \frac{1}{2}$, 1, and 1 amperes. The heats developed are, by the rule, in the ratio $100 \times \frac{1}{2} : 35 \times 1$ and 50×1 or 50 : 35 : 50.

The same problem can be done directly by Rule 31, thus: The three lamps develop heat in the ratio $\frac{100^2}{200} : \frac{35^2}{35} : \frac{50^2}{50} = 50 : 35 : 50$. This is the direct and preferable method of calculation.

Note.—For "heat," "rate of energy," or "rate of work" can be read in these rules.

THE JOULE OR GRAM-CALORIE.

The last rules and problems only touch upon the *relative proportions* of heat; they do not give any actual quantity. If we can express in units of the same class a standard quantity of heat, then by determining the relation of the standard to any other quantity, we arrive at a real quantity. Such a standard is the joule, sometimes called the "calorie" or "gram-calorie." A joule is the quantity of heat required to raise the temperature of 1 gram of water 1 degree centigrade. It is often expressed as a water-gram-degree C. or w. g. d. C. or for shortness g. d. C., from the initials of the factors. It is unfortunate that it is called the calorie as the name is common to the water-kilogram-degree C. or kg.d. C. The joule is equal to 4.16×10^7 or 41,600,000 ergs.

It will be remembered that practical electric units are based on multiples of the C. G. S. units of which the erg is one. The joule comes in the C. G. S. order. Therefore to determine quantities of heat the following is a general rule when the practical electric units are used.

Rule 33. Obtain the expression of rate of heat developed, or of rate of energy, or of rate of work in volt amperes. Reduce to C. G. S. units (ergs) by multiplying by 10^7 and divide by the value of a joule in ergs (4.16×10^7). The quotient is joules or water-gram degrees C. per second.

$$Q = \frac{E \times C \times 10^7}{4.16 \times 10^7}$$

EXAMPLE.

A current of 20 amperes is flowing through a wire. What heat is developed in a section of the wire whose ends differ in potential by 110 volts?

Solution: The rate of energy in watts or volt-amperes = $110 \times 20 = 2200$. In the C. G. S. units this is expressed by $(110 \times 10^8) \times (20 \times 10^{-1}) = 2200 \times 10^7$ ergs. per second; $\therefore$ quantity of heat = $2200 \times 10^7 \div 4.16 \times 10^7 = 528.8$ joules or gram-degrees-centigrade per second.

As $10^7 \div 10^7 = 1$ the rule can be more simply stated thus:

Rule 34. The quantity of heat produced per second in a circuit by a current is equal to the product of the watts by $\frac{1}{4.16}$ or by .24.

$$Q = 0.24\, C\, E. \text{ or } 0.24 \frac{E^2}{R}$$

EXAMPLES.

A difference of potential of 5.5 volts is maintained at the terminals of a wire of $\frac{1}{10}$ ohm resistance. How many joules per second are developed?

Solution: By Ohm's law, current = $5.5 \div \frac{1}{10} = 55$ amperes. By the rule $55 \times 5.5 \times 0.24 = 72.6$ joules per second.

Note.—The energy of a current is given by Rules 30, 31 and 32 in watts, so that all cases are provided for by a combination of one or the other of these rules with Rule 34. An example will be given for each case.

A current of .8 ampere is sent through 50 lamps in series, each of $137\frac{1}{2}$ ohms resistance. What heat does it develop per second?

Answer: The resistance $= 137\frac{1}{2} \times 50$. By rules 30 and 34 we have, rate of heat produced $= .8^2 \times 137\frac{1}{2} \times 50 = 4400$ watts. $4400 \times 0.24 = 1056$ joules per second.

Rules 31 and 34. Fifty incandescent lamps, 110 volt, $137\frac{1}{2}$ ohms, each are placed in parallel. What heat per second do they develop?

Solution: By Ohm's law total resistance $= 137\frac{1}{2} \div 50 = 2.75$ ohms. By rules 31 and 34 rate of heat produced $= 110^2 \div 2.75 = 4400$ watts and $4400 \times 0.24 = 1056$ joules per second as before.

Rules 32 and 34. Through 50 incandescent 110 volt lamps a current of .8 ampere is passed, the lamps being in series. What heat per second do they develop?

Solution: By rules 32 and 34 rate of heat $= 110 \times 50 \times .8 = 4400$ watts and 1056 joules per second as before.

These three examples are purposely made to refer to the same set of lamps, to show that rules 30, 31, and 32 are identical. Each fits one of the three forms of statement of data. They also are designed to bring out the fact that the unit "watts," being based partly on amperes, includes the idea of rate, not of absolute quantity. Hence watts "per second" is not stated, as it would be meaningless or

redundant, while the joule, denoting an absolute quantity, has to be stated "per second" to indicate the rate. There is such a unit as an "ampere-second," viz., the "coulomb," but there is no such thing as an "ampere per second," or if used it means the same as an "ampere per hour," "ampere per day" or "ampere." The same applies to watts and to power units such as "horse-power."

Specific Heat.

The specific heat of a substance is the ratio of its capacity for heat to that of an equal quantity of water. It almost invariably is referred to equal weights. Here it will be treated only in that connection.

The coefficient of specific heat of any substance is the factor by which the specific heat of water (= 1 or unity) being multiplied the specific heat of the substance is produced.

Rule 35. The weight of any substance corresponding to any number of joules multiplied by its specific heat gives the corresponding weight of water, and *vice versa*.

Example.

A current of .75 amperes is sent for 5 minutes through a column of mercury whose resistance was 0.47 ohm. The quantity of mercury was 20.25 grams, and its specific heat 0.0332. Find the rise of temperature, assuming that no heat escapes by radiation. (*Day.*)

Solution: By Rules 30 and 34, we find rate of heat $= .75^2 \times .47 = .264375$ watts; $.264375 \times .24 = .06345$ joules per second. The current is to last for 300 seconds ∴ total joules $= .06345 \times 300 = 19.035$ joules. This must be divided by the specific heat of mercury to get the corresponding weight of mercury; $19.035 \div .0332 = 573$ gram degrees of mercury. Dividing this by the weight of mercury, 20.25 grams, we have $573 \div 20.25 = 28°$ C.

Rule 36. By radiation and convection, $\frac{1}{4000}$ joule about is lost by any unpolished substance in the air for each square centimeter of surface, and for each degree that it is heated above the atmosphere.

EXAMPLE.

A conductor of resistance, 8 ohms, has a current of 10 amperes passing through it. It is 1 centimeter in circumference, and 100 meters long. How hot will it get in the air?

Solution: By Rule 30, etc., the heat developed per second in joules is $10^2 \times 8 \div 4.16 = 192.3$ joules. The surface of the conductor in centimeters is $10,000 \times 1 = 10,000$ sq. cent. It therefore develops heat at the rate of $192.3 \times 10^{-4} = .01923$ joules per second per square centimeter of surface. When the loss by radiation and convection is equal to this, it will remain constant in temperature. Therefore $.01923 \div \frac{1}{4000} = 76.92$, the number of degrees C. above the air to which the conductor could be heated by such a current.

Results of this character are only approximate, as the coefficient, $\frac{1}{1000}$, is not at all accurate.

Rule 37. The cube of the diameter in centimeters of a wire of any given material that will attain a given temperature centigrade under a given current is equal to the product of the square of the current in amperes × Sp. Resistance in microhms of the substance of the wire, multiplied by .000391, and divided by the temperature in degrees centigrade.

$$d^3 = \frac{C^2 \times \text{Sp. Res.} \times .000391}{t^\circ}$$

EXAMPLES.

A lead safety catch is to be made for a current of 7.2 amperes. Its melting point is 335° C., and its specific resistance 19.85 microhms per cubic centimeter. What should its diameter be? (*Day.*)

Solution: By the rule, the cube of the diameter = $7.2^2 \times 19.85 \times .00039 \div 335 = .001198$. The cube root of this gives the diameter in centimeters. It is .10582 or .106 C.C.

A copper wire is to act as safety catch for 500 amperes: melting point 1050° C—Sp. Resistance 1.652 microhms. What should its diameter be? (*Day.*)

Solution: Cube of diameter $= \frac{500^2 \times 1.652 \times .000391}{1050} = \frac{159{,}418}{1050} = .1523$. The cube root of this is .5341 centimeter, the thickness of the wire sought for.

It will be observed that in this rule no attention is paid to the length of the wire, as the effect of a current in melting a wire has no reference to its

length. The time of fusion will vary with the specific heat, but will, of course, be only momentary.

WORK OF A CURRENT.

Rule 38. The work of a current in a given circuit is proportional to the volt amperes. $W = EC$

EXAMPLE.

A current A of 3.5 amperes with difference of potential in the circuit of 20 volts is to be compared to B, a 3 ampere current with a difference of potential of 1000 volts in the circuit; what is the ratio of work produced in a unit of time?

Solution: Work of A : work of B :: $3.5 \times 20 : 3 \times 1000$ or as $70 : 3000$ or as $1 : 428\frac{5}{10}$.

Rule 39. The work of a current in a given circuit is equal to the volt-coulombs divided by the acceleration of gravitation (9.81 meters). This gives the result in kilogram-meters. (7.23 foot pounds.) $W = \frac{E.\,C.\,t}{9.81}$

EXAMPLE.

A current of 20 amperes is maintained in a circuit by an E. M. F. of 20 volts. What work does it do in one minute and a half (90 sec.)?

Solution: Work $= 20 \times 20 \times 90$ sec $\div 9.81 = 3670$ kgmts. and $3670 \times 7.23 = 26{,}534$ foot pounds.

Note.—This is easily reduced to horse-power. 26,534 foot pounds in 90 sec. = 17,688 foot pounds in 1 min. 1 H. P. = 33,000 foot pounds per min. ∴ $\frac{17688}{33000} = .536$ H. P. of above current and circuit. The same result can be obtained by Rule 41 thus: $\frac{20 \times 20}{746} = .536$ H. P.

Rule 40. To determine work done by a current in a given circuit apply Rules 30, 31 or 32 as the case requires. These give directly the watts. Multiply by seconds and divide by 9.81. The result is kilogram-meters.

EXAMPLES.

10 amperes are maintained for 55 sec. through 15 ohms. What is the work done?

Solution: By rule 32, watts $= 10^2 \times 15 = 1500$. Work $= 1500 \times 55 \div 9.81 = 8409$ kgmts.

1000 volts are maintained between terminals of a lead of 20 ohms resistance. Calculate the work done per hour.

Solution: By Rule 32 watts $= 1000^2 \div 20 = 50,000$. One hour $= 3600$ sec. Work $= 50,000 \times 3600 \div 9.81 = 18,348,623$ kgmts.

These rules give the basis for determining the expense of maintaining a current. The expense of maintaining a horse-power or other unit of power or work being known the cost of electric power is at once calculable.

ELECTRICAL HORSE-POWER.

Power is the rate of doing work or of expending energy. In an electric circuit one horse-power is equal to such a product of the current in amperes, by the E. M. F. in volts as will be equal to 746.

Rule 41. The electric horse power is found by multiplying the total amperes of current by the volts or E. M. F. of a circuit or given part of one and dividing by 746.

$$H.P. = \frac{EC}{746}$$

EXAMPLE.

250 incandescent lamps are in parallel or on multiple arc circuit. Each one is rated at 110 volts and 220 ohms. What electric H. P. is expended on their lighting?

Solution: The resistance of all the lamps in parallel is equal to $\frac{220}{250}$ ohm. The current is equal to $110 \div \frac{220}{250} = 125$ amperes. H. P. $= 125 \times 110 \div 746 = 18\frac{4}{10}$ H. P. or $13\frac{6}{10}$ lamps to the electrical H. P.

As it is a matter of indifference as regards absorption of energy how the lamps are arranged, a simpler rule is the following, where horse-power required for a number of lamps or other identical appliances is required.

Rule 42. Multiply together the voltage and amperage of a single lamp or appliance; multiply the product by the number of lamps or appliances and divide by 746.

EXAMPLE.

Take data of last problem and solve it.

Solution: Current of a single lamp $= \frac{110}{220} = \frac{1}{2}$ ampere. H. P. $= 110 \times \frac{1}{2} \times 250 \div 746 = 18\frac{4}{10}$ H. P.

When the voltage and amperage are not given directly, the missing one can always be calculated by Ohm's law and the above rules can then be applied. The same can be done by applying following:

Rule 43. To determine the electrical horse-power apply Rules 30, 31, or 32; these give directly the watts; multiplying the result by $\frac{1}{746}$ or dividing by 746 gives the horse-power.

EXAMPLES.

A current of 10 amperes is maintained through 50 ohms resistance. What is the electrical horse-power?

Solution: By rules 30 and 43 we have watts = $10^2 \times 50 = 5000$ and electrical horse-power = $5000 \div 746$ or 6.7 H. P.

An electromotive force of 1500 volts is maintained in a circuit of 200 ohms resistance. What is the electrical horse-power?

Solution: By Rules 31 and 43 we have watts = $1500^2 \div 200 = 11{,}250$. Electrical horse-power = $11{,}250 \div 746$ or 15 H. P. (nearly).

Thus the volt-amperes or watts are units of rate of heat energy or of rate of mechanical energy. The ratio of joules per second to a horse-power is 746 : 4.16 or 179.3 joules per second = 1 H. P. Other ratios of power and heat units will be found in the tables.

DUTY AND EFFICIENCY OF ELECTRIC GENERATORS.

Rule 44. The duty of an electric generator is the quotient obtained by dividing the total electric energy by the mechanical energy expended in turning the armature.

$$D = \frac{\text{e. H. P. (total)}}{\text{m. H. P.}}$$

EXAMPLES.

A dynamo is driven by the expenditure of 58 H. P. Its internal resistance is 10.7 ohm. The

resistance of the outer circuit is 150 ohms and it maintains a current of 16 amperes. What is its duty?

Solution: The total electrical H. P. is found by Rules 30 and 43 to be $16^2 \times 160.7 \div 746 = 55.1$ H. P. Duty $= 55.1 \div 58.0 = 95\%$.

The result must always be less than unity; if it exceeded unity it would prove that there had been an error in some of the determinations.

Rule 45. The commercial efficiency of a generator is the quotient obtained by dividing the electric energy in the outer circuit by the mechanical energy expended in turning the armature.

$$\text{C. Eff.} = \frac{\text{e. H. P. (outer circuit)}}{\text{m. H. P.}}$$

EXAMPLES.

What is the commercial efficiency of the dynamo just cited?

Solution: The electrical H. P. of the outer circuit is found by the same rules to be $16^2 \times 150 \div 746 =$ 51.5 commercial efficiency $= 51.5 \div 58.0 = 88.8\%$.

Rule 46. The resistance of the outer circuit is to the total resistance, as the commercial efficiency is to the duty.

EXAMPLES.

Take the case of the generator last given and from its duty calculate the commercial efficiency.

Solution: $150 : 160.7 :: x : 95.0 \therefore x = 88.8$ or 88.8%.

CHAPTER VIII.

BATTERIES.

General Calculations of Current.

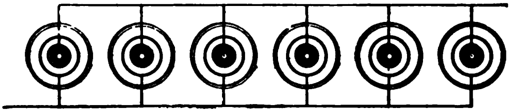
A battery is rated by the resistance and electro-motive force of a single cell, which factors are termed the cell constants. In the case of storage batteries, whose susceptibility to polarization is very slight, the resistance is often assumed to be neglible. It is not so, and in practice is always knowingly or otherwise allowed for.

From the cell constants its energy-constant may be calculated by Rule 31, as equal to the square of its electro-motive force divided by its resistance. This expresses its energy in watts through a circuit of no resistance.

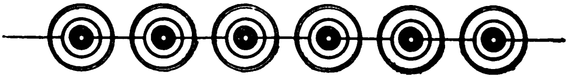
There are two resistances ordinarily to be considered, the resistance of the battery which is designated by R or by n R if the number of cells is to be implied and the resistance of the external circuit which is designated by r.

Rule 47. The current given by a battery is equal to its electro-motive force divided by the sum of the external and internal resistances.

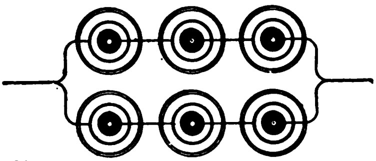
$$C = \frac{E}{R + r}$$



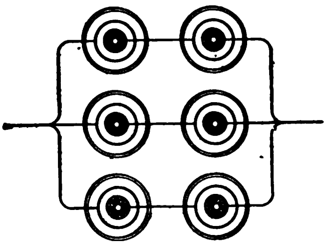
Six cells in parallel.



Six cells in series.



Six cells—two in parallel, three in series.



Six cells—three in parallel, two in series.

ARRANGEMENT OF BATTERY CELLS.

Example.

A battery of 50 cells arranged to give 75 volts E. M. F. with an internal resistance of 100 ohms sends a current through a conductor of 122 ohms resistance. What is the strength of the current?

Solution: Current $= 75 \div (100 + 122) = .338$ ampere. This rule has already been alluded to under Ohm's law (page 14).

Arrangement of Cells in Battery.

In practise the cells of a battery are arranged in one of three ways. *a:* All may be in series; *b:* all may be in parallel; *c:* some may be in series and some in parallel, so as to represent a rectangle, *s* cells in series by *p* cells in parallel, the total number of cells being equal to the product of *s* and *p*.

Other arrangements are possible. Thus the cells may represent a triangle, beginning with one cell, followed by two in parallel and these by three in parallel and so on. This and similar types of arrangement are very unusual and little or nothing is to be gained by them.

Rule 48. The electromotive force of a battery is equal to the E. M. F. of a single cell multiplied by the number of cells in series.

Rule 49. The resistance of a battery is equal to the number of its cells in series, multiplied by the resistance of a single cell and divided by the number of its cells in parallel.

$$R._{\text{battery}} = \frac{s\,R}{p}$$

EXAMPLES.

A battery of 50 gravity cells 1 volt, 3 ohms each is arranged 10 in parallel and 5 in series. What is its resistance and electromotive force?

Solution: Resistance $= 5 \times 3 \div 10 = 1.5$ ohms.
E. M. F. $= 5 \times 1 = 5$ volts.

The same battery is arranged all in parallel; what is its resistance and E. M. F.?

Solution: This gives one cell in series.
Resistance $= 1 \times 3 \div 50 = .06$ ohms.
E. M. F. $= 1 \times 1 = 1$ volt.

The same battery is arranged all in series; what is its resistance?

Solution: This gives one cell in parallel.
Resistance $= \frac{50 \times 3}{1} = 150$ ohms.
E. M. F. $= 50 \times 1 = 50$ volts.

The current given by a battery is obtained from these rules and from Ohm's law.

EXAMPLE.

150 cells of a battery (cell constants 1.9 volts, ½ ohm) are arranged 10 in series and 15 in parallel. They are connected to a circuit of 1.7 ohms resistance. What is the current?

Solution: The resistance of the battery $= \frac{10 \times \frac{1}{2}}{15} =$ .333 ohms. The E. M. F. $= 10 \times 1.9 = 19$ volts. Current $= 19 \div (.333 + 1.7) = 9.34$ amperes.

Cells Required for a Given Current.

To calculate the cells required to produce a given current through a given resistance and the arrangement of the cells proceed as follows.

Rule 50. Calculate the cell current through zero external resistance. Case A. If it is twice as great or more than twice as great as the current required apply Rule 51. Case B. If less than twice as great and more than equal or less than equal and more than one half as great as the current required and so on apply Rule 52. Case C. If the cell current is equal to or is a unitary fraction ($\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, etc.) of the current required apply Rule 53.

Rule 51. Case A. Divide the required difference of potential of the outer circuit by the voltage of a single cell diminished by the product of the required current multiplied by the resistance of a single cell. Arrange the cells in series.

$$N = \frac{e}{E - CR}$$

Examples.

Five lamps in parallel, each of 100 volts 200 ohms, are to be supplied by a battery whose cell constants are 2 volts $\frac{1}{5}$ ohm. How many cells and what arrangement are required?

Solution: Cell current $= \frac{2}{\frac{1}{5}} = 10$ amperes. The resistance of the five lamps in parallel (Rule 12) $= \frac{200}{5} = 40$ ohms. The required current therefore $= \frac{100}{40} = 2\frac{1}{2}$ amperes. As 10 exceeds $2\frac{1}{2} \times 2$ (Rule 50) it falls under case A. By Rule 51 the number of cells is $\frac{100}{2 - (2\frac{1}{2} \times \frac{1}{5})} = \frac{100}{\frac{1}{5}} = 66.6$ or 67 cells, as a cell cannot be divided. The cells must be in series.

Proof: The E. M. F. of the 67 cells in series $= 67 \times 2 = 134$ volts; their resistance $= 67 \times \frac{1}{5} =$ 13.4 ohms. The resistance of the lamps in parallel is 40 ohms. Hence by Ohm's law the current $= \frac{134}{40 + 13.4} = 2.51$ amperes, the current required.

The same lamps are placed in series. Calculate the cells of the same battery required. Cell current = 10 amperes. Current required $= \frac{100 \times 5}{200 \times 5}$ or $\frac{1}{2}$ ampere. As 10 exceeds $\frac{1}{2} \times 2$ (Rule 50) it falls again under case A. By Rule 51 cells required $= \frac{500}{2 - (\frac{1}{2} \times \frac{1}{5})}$ $= 263$.

Proof: Current $= \frac{526}{52.6 + 1000} = \frac{1}{2}$ ampere the current required. 526 is the number of cells multiplied by the voltage of one cell; 52.6 is the number of cells multiplied by the resistance in ohms of a single cell; 1000 is the resistance of a single lamp, 200 ohms, multiplied by the number, 5, of lamps in series.

Whenever the arrangement and number of cells of a battery has been calculated the calculation should be proved as above.

Rule 52. Case B. Group two or more cells in parallel so as to obtain by calculation from them through no external resistance a current twice as great or more than twice as great as the required current. Then treating the group as if it was a single cell apply Rule 51 to determine the number of groups in series.

EXAMPLE.

Assume the same lamps in parallel, requiring the

current already calculated of $2\frac{1}{2}$ amperes. Assume a battery of constants 1 volt .25 ohm, giving a cell current of 4 amperes. This is less than $2\frac{1}{2} \times 2$ and more than $2\frac{1}{2} \times 1$; therefore it falls under Case B.

Solution: A group of two cells in parallel gives $\frac{1}{.125} = 8$ amperes. 8 exceeds $2\frac{1}{2} \times 2$ $\therefore$ applying Rule 49 we have number of groups $= 100 \div [1 - (2\frac{1}{2} \times .125)] = 146$ groups in series. Total number of cells $= 2$ in parallel, 146 in series $= 292$ cells.

Proof: Current $= 146 \div (40 + 18.25) = 2.5$ amperes.

Rule 53. Case C. Place as many cells in series as will give twice the required voltage. Place as many cells in parallel as will give a resistance equal to that of the external circuit.

EXAMPLE.

Assume the same lamps in parallel. Assume a battery of cell constants, 1 volt, 4 ohms. The lamp current is $2\frac{1}{2}$ amperes. The cell current is $\frac{1}{4}$ ampere. The cell current therefore equals $(\frac{1}{4} \div 2\frac{1}{2})$ $\frac{1}{10}$ of the required current. This falls under Case C. and is solved by Rule 53.

Solution: Voltage required 100. By the rule cells in series $= 100 \times 2 = 200$. These have a resistance of 800 ohms. To reduce this to the resistance of the outer circuit, viz., 40 ohms, $800 \div 40 = 20$ cells must be placed in parallel. Total cells $= 20 \times 200 = 4000$ cells.

Proof: Current = 200 ÷ (40 + 40) = 2.5.

Rule 54. All cases coming under Case C. may be simply solved for the total number of cells by dividing the external energy by the cell energy and multiplying by 4. This gives the number of cells.

EXAMPLE.

Take as cell constants .75 volt 1½ ohm giving ½ ampere. Assume 20 lamps, each 50 volts, 50 ohms and 1 ampere. As ½ ÷ 1 is a unitary fraction (½) Case C. applies.

Solution: Cell energy = ½ × .75 = .375 watts. External energy = 50 × 1 × 20 = 1000 watts. (1000 ÷ .375) × 4 = 10,666 cells.

Solution by Rule 53: Voltage required taking lamps in series = 20 × 50 = 1000. To give twice this voltage requires 2000 ÷ .75 = 2667 cells in series whose resistance is 2667 × 1.5 = 4000 ohms. To reduce this to 1000 ohms we need 4 such series of cells in parallel giving 10,668 cells.

Proof: Current $= \frac{2667 \times .75}{(\frac{2667}{4} \times 1\frac{1}{2}) + 1000} = 1$ ampere.

Slight discrepancies will be noticed in the current strength given by different calculations. This is unavoidable as a cell cannot be fractioned or divided.

EFFICIENCY OF BATTERIES.

Rule 55. The efficiency of a battery is expressed by dividing the resistance of the external circuit by the total resistance of the circuit.

$$\text{Efficiency} = \frac{R}{R + r}$$

EXAMPLE.

A battery consists of 67 cells in series of constants 2 volts $\frac{1}{5}$ ohm. It supplies 5 lamps in parallel, each 100 volts 200 ohms constants. What is its efficiency?

Solution: The resistance of the battery is $67 \times \frac{1}{5} = 13.4$ ohms. The resistance of the lamps is (Rule 12) $\frac{200}{5} = 40$ ohms. Therefore the efficiency of the battery is $40.0 \div (40 + 13.4) = .749$ or 74.9%.

Rule 56. To calculate the number of battery cells and their arrangement for a given efficiency: Express the efficiency as a decimal, multiply the resistance of the external circuit by the complement of the efficiency (1—efficiency) and divide the product by the efficiency; this gives the resistance of the battery. Add the two resistances and multiply their sum by the current to be maintained for the E. M. F. of the battery. Arrange the cells accordingly as near as possible to these requirements.

EXAMPLES.

Five lamps, each 100 volts 200 ohms in parallel are to be supplied by a battery of cell constants 2 volts .4 ohm. The efficiency of the battery is to be as nearly as possible 75%. Calculate the number of cells and their arrangement.

Solution: The constants of the external circuit are 40 ohms (Rule 12) and 100 volts. Applying the rule we have $[40 \times (1—.75)] \div .75 = \frac{1000}{75} = 13\frac{1}{3}$ ohms, the resistance of the battery. By Ohm's law the E. M. F. of the battery $= (40 + 13\frac{1}{3}) \times 2.5 =$

133⅓ volts. These constants, 13⅓ ohms and 133⅓ volts, require 67 cells in series and 2 in parallel.

Proof: *a.* Of efficiency, by Rule 55, $\frac{40}{40+13\frac{1}{3}} = .75$ or 75%. *b.* Of number of cells and of their arrangement $67 \times 2 = 134$ volts; $(67 \times .4) \div 2 = 13.4$ ohms; $134 \div (13.4 + 40) = 2.5$ amperes.

Rule 57. Where a fractional or mixed number of cells in parallel are called for to produce a given efficiency, take a group of the next highest integral number of cells in parallel and proceed as in Rule 51.

EXAMPLE.

Assume a current of 3⅓ amperes to be supplied through a resistance of 30 ohms, absorbing 100 volts E. M. F. Let the cell constants of a battery to supply this circuit be 2 volts, ⅙ ohm. Calculate the cells and their arrangement for 80 per cent. efficiency.

Solution: By Rule 56 efficiency = .80 and $\frac{30 \times (1-.80)}{.80} = 7\frac{1}{2}$ ohms, which is the required resistance of the battery; $7\frac{1}{2} + 30 = 37\frac{1}{2}$ ohms are the total resistance of the circuit. By Ohm's law, $37\frac{1}{2} \times 3\frac{1}{3} = 125$ volts, the required E. M. F. of the battery. This requires 63 cells in series, with a resistance for one series of $63 \times \frac{1}{6} = 10\frac{1}{2}$ ohms. To reduce this to $7\frac{1}{2}$ ohms $\frac{10\frac{1}{2}}{7\frac{1}{2}} = 1.4$ cells in parallel are required. As this is a mixed number we take the next highest integral number and place 2 cells in parallel. The constants of this group of 2 cells are 2 volts, $\frac{1}{12}$

ohm. Applying Rule 51 we have for the number of such groups in series; $\frac{100}{2-(3\frac{1}{3} \times \frac{1}{12})} = 58$ groups in series. As there are 2 cells in parallel the total cells are 116, of resistance, $58 \times \frac{1}{12} = 4.83$ ohms, and of E. M. F., $58 \times 2 = 116$ volts.

Proof: Of efficiency by Rule 55, $\frac{30}{30 + 4.83} = 86.1\%$. Of number and arrangement of cells $\frac{116}{30 + 4.83} = 3.33$ amperes.

It is to be observed that the efficiency thus obtained is far from what is required. In most cases accuracy can only be attained by arranging the battery irregularly, which is unusual in practice. An example will be found in a later chapter.

Chemistry of Batteries.

One coulomb of electricity will set free .010384 milligrams of hydrogen. The corresponding weights of other elements or compounds are found by multiplying this factor by the chemical equivalent, and dividing by the valency of the element or metal of the base of the compound in question.

An element or other substance in entering into any chemical combination develops more or less heat, always the same for the same weight and combination. The atomic weight of an element or the molecular weight of a compound divided by the valency of the element or metal of its base gives the original chemical equivalent.

The quantities of heat evolved by the combination of quantities of substances expressed by their original chemical equivalents multiplied by one gram are termed the thermo-electric equivalents of the elements or substances in question. In the tables it is expressed in kilogram degrees C. of water (kilogram-calories).

From the thermo-electric equivalent of a combination we find the volts evolved by it or absorbed by the reciprocal action of decomposition.

Rule 58. The volts evolved by any chemical combination or required for any chemical decomposition are equal to the thermo-electric equivalent in kilogram-calories multiplied by .043.

$$E = .043 \times H.$$

EXAMPLES.

What number of volts is required to decompose water?

Solution: From the table we find that the combination of one gram of hydrogen with eight grams of oxygen liberates 34.5 calories. Then $34.5 \times .043 = 1.48$ volts.

Rule 59. To determine the voltage of a galvanic couple subtract the kilogram calories corresponding to decompositions in the cell from those corresponding to combinations in the cell for effective energy and multiply by .043 for volts.

EXAMPLES.

Calculate the voltage of the Smee couple.

Solution: In this battery zinc combines with

oxygen, giving out 43.2 calories and combines with sulphuric acid, giving 11.7 more calories; a total of 54.9 calories. An equivalent amount of water is at the same time decomposed acting as counter-energy of 34.5 calories. The effective energy is 54.9 − 34.5 = 20.4 calories. The voltage = 20.4 × .043 = .877 volts.

Calculate the voltage of the sulphate of copper battery.

Solution: Here we have combination of zinc with sulphuric acid as above 54.9 calories; decomposition of copper sulphate 19.2 + 9.2 = 28.4 ∴ 54.9 − 28.4 = 26.5 calories effective energy 26.5 × .034 = 1.14 volts.

It will be noticed that these results are approximate. Some combinations are omitted in them as either of unknown energy, or of little importance.

Work of Batteries.

The rate of work of a battery is proportional to the current multiplied by the electro-motive force. The work is distributed between the battery and the external circuit in the ratio of their resistances as by Rule 55. The horse-power, and heating power are calculated by Rules 30–43, care being taken to distribute the energy acording to the resistance by the following rule:

Rule 60. The effective rate of work or the rate of work in the external circuit of a battery, is equal to the

total rate multiplied by the efficiency of the battery expressed decimally.

EXAMPLE.

25 cells of 2 volt 1 ohm battery are arranged in series on an external circuit of 250 ohms resistance. What work do they do in that circuit?

Solution: The current (Ohm's law) $= \frac{25 \times 2}{25 + 250} = \frac{2}{11}$ 1.818 amperes. Total rate of work $= 1.818 \times 50$ volts $= 90.9$ watts. Efficiency of battery $= \frac{250}{275} = 90$ per cent. (nearly). Effective rate of work $= 1.818 \times 50 \times .90 = 81.81$ watts.

CHEMICALS CONSUMED IN A BATTERY.

Rule 61. The chemicals consumed in grams by a battery for one kilogram-meter (7.23 foot lbs.) of work are found by multiplying the combining equivalent of the chemical by the number of equivalents in the reaction by the constant .000101867 and dividing by the product of the E. M. F. by the valency of the element in question.

$$W = \frac{\text{Equiv.} \times n \times .000101867}{E \times \text{valency}}$$

EXAMPLES.

What is the consumption of zinc and sulphate of copper per kilogram-meter of work in a Daniel's battery?

Solution: Take the E. M. F. as 1.07 volt. The equivalent of zinc, a dyad, is 65 and one atom enters into the reaction. The zinc consumed. therefore $= \frac{65 \times 1 \times .000101867}{1.07 \times 2} = .00309$ gram.

The equivalent of copper sulphate, is 159.4. One equivalent enters into the reaction carrying with it one atom of the dyad metal copper. The weight consumed therefore $= \frac{159.4 \times 1 \times .000101867}{1.07 \times 2} = .0076$ grams. Add 56.46% for water of crystallization.

All these quantities are for one kilogram-meter of work (7.23 foot lbs.) which may be more or less effective according to circumstances as developed in Rules 44, 45, and 60.

Decomposition of Compounds by the Battery.

In cases where a compound has to be decomposed by a battery two resistances may be opposed to the work. One is the ohmic resistance of the solution, which is calculated by Rule 16. The other is the electromotive force required to decompose the solution. This is best treated as a counter-electromotive force. Then from the known data the current rate is calculated, and from the electro-chemical equivalents the quantity of any element deposited by a given number of coulombs is determined.

Rule 62. To calculate the metal or other element liberated by a given current per given time proceed as follows: Calculate the resistance. Determine the counter-electromotive force of the solution by Rule 58 and subtract it from the E. M. F. of the battery or generator. Apply Ohm's law to the effective voltage thus determined and to the calculated resistances to find the current. Multiply the electro-chemical equivalent of the element by the coulombs or ampere-seconds.

EXAMPLE.

A bath of sulphate of copper is of specific resistance 4 ohms. The electrodes are supposed to be 10,000 sq. centimeters in area and 5 centimeters apart. Two large Bunsen elements in series of 1.9 volts .12 ohms each are used. What weight in milligrams of copper will be deposited per hour?

Solution: By Rule 16 the resistance of the solution is $\frac{46 \times 5}{10000} = 0.023$. The electro-chemical equivalent of copper is .00033 grams. The thermo-electric equivalent for copper from sulphate of copper is $19.2 + 9.2 = 28.4$ calories. The E. M. F. corresponding thereto $= 28.4 \times .043 = 1.22$ volts counter E. M. F. The E. M. F. of the battery $= 1.9 \times 2 = 3.8$ volts, giving an effective E. M. F. of $3.8 - 1.22 = 2.58$ volts. The resistance of the battery $= .12 \times 2 = .24$ ohms. The current $= \frac{2.58}{.24 + .023} = 9.8$ amperes. This gives per hour $9.8 \times 3,600 = 35,280$ coulombs, and for copper deposited $.00033 \times 35,280 = 11.64$ grams.

In many cases one electrode is made of the material to be deposited and being connected to the carbon end of the battery or generator is dissolved as fast as the metal is deposited. In such case there is no counter electro-motive force to be allowed for.

EXAMPLE.

Take the last case and assume one electrode (the anode) to be of copper and to dissolve. Calculate the deposit.

Solution: Current $= 3.8 \div (.24 + .023) = 14.4$ amperes $= 51,840$ coulombs per hour $= .00033 \times 51,840 = 17.10$ grams of copper.

CHAPTER

ELECTRO–MAGNETS, DYNAMOS AND MOTORS.

THE MAGNETIC FIELD AND LINES OF FORCE.

A CURRENT of electricity radiates electro-magnetic wave systems, and establishes what is known as a field of force. The field is more or less active or intense according to the force establishing it. The intensity of a field is for convenience expressed in LINES OF FORCE. These are the units of magnetic intensity, often called units of magnetic flux, and the line as a unit is comparable to the ampere × 10, which is the C. G. S. unit of current. A line of force is that quantity of magnetic flux which passes through every square centimeter of normal cross-section of a magnetic field of unit intensity. The line is at right angles to the plane of normal cross-section of such field. Such intensity of field exists at the center of curvature of an arc of a circle of radius 1 centimeter, and whose length is 1 centimeter, when a current of 10 amperes passes through this arc. Practically it is the amount which passes through an area of one square centimeter, situated in the center of a circle 10 centimeters in diameter,

surrounded by a wire through which a current of 7.9578 amperes is passing. The plane of the circle is a cross-sectional plane of the field; a line perpendicular to such plane gives the direction of the lines of force, or of the magnetic flux.

This cross-sectional area is often spoken of as the field of force. As a field exists wherever there are lines of force, there are in each magnetic circuit either an infinite number of fields of force, or a field of force is a volume and not an area.

The number of lines of force or of magnetic flux per unit cross-sectional area of the magnetic circuit, i. e. per unit area of magnetic field, expresses the intensity of the field. In soft iron, it may run as high as 20,000 or more lines per square centimeter of cross-section of the iron which is magnetized.

Just as we might speak of a bar of copper acting as conductor for 20,000 C. G. S. units of current, or 2000 amperes, so we may speak of the iron core of a magnet carrying 20,000 lines of force.

Permeance and Reluctance.

This action of centralizing in its own material lines of force is analogous to "conductance." It is termed PERMEANCE. Its reciprocal is termed RELUCTANCE, which is precisely analagous to "resistance." Iron, nickel, and cobalt possess high permeance; the permeance of air is taken as unity. At a low degree of magnetization, soft iron pos-

sesses 10,000 times the permeance of air. At high degrees of magnetization, it possesses much less in comparison with air, whose permeance is unchanged under all conditions.

There is no substance of infinitely high reluctance, which is the same as saying that there is no insulator of magnetism.

Magnetizing Force and the Magnetic Circuit.

The producing cause of the magnetic flux or magnetization just described is in practice always a current circulating around an iron core. The name of MAGNETIZING FORCE is often given to it. It is the analogue of electro-motive force, and is measured by the lines of force it establishes in a field of air of standard area.

A high value for the magnetic force is 585 lines per square centimeter. It is proportional to the amperes of current and to the number of turns the conductor makes around it. Its intensity is often given in ampere-turns.

Magnetization always implies a circuit. As far as known, magnetic lines of force cannot exist without a return circuit, exactly like electric currents. But owing to the imperfect reluctance of all materials, the lines of force can complete their circuit through any substance. In a bar magnet the return branch of the circuit is through air.

In the same magnetic circuit, the planes of normal cross-section lie at various angles with each other.

The law of a magnetic circuit is exactly comparable to Ohm's law. It is as follows:

Rule 63. The magnetization expressed in lines of force is equal to the magnetizing force divided by the reluctance or multiplied by the permeance of the entire circuit.

This rule would be of very simple application, except for the fact that reluctance increases, or permeance decreases, with the magnetization, and the rate of variation is different for different kinds of iron.

Rule 64. Permeability is the ratio of magnetization to magnetizing force, and is obtained by dividing magnetization by magnetizing force.

Permeability has to be determined experimentally for each kind of iron. It is simply the expression of a ratio of two systems of lines of force. It always exceeds unity for iron, nickel, and cobalt. The specific susceptibility of any particular iron to magnetization is its permeability. The susceptibility of a portion, or of the whole of a magnetic circuit is its permeance.

General Rules for Electro-Magnets.

The traction of a magnet is the weight it can sustain when attached to its armature. It is pro-

portional to the square of the number of lines of force passing through the area of contact.

Rule 65. The traction of a magnet in pounds is equal to the square of the number of lines of force per square inch, multiplied by the area of contact and divided by 72,134,000. In centimeter measurement the traction in pounds is equal to the square of the number of lines of force per square centimeter multiplied by the area of contact and divided by 11,183,000. The traction in grams is equal to the latter dividend divided by 24,655 ($8\pi \times 981$); for dynes of traction the divisor is 25.132 (8π)

EXAMPLES.

A bar of iron is magnetized to 12,900 lines per square inch; its cross-section is 3 square inches. What weight can it sustain, assuming the armature not to change the intensity of magnetization?

Solution: $12{,}900^2 \times 3 = 499{,}230{,}000$. This divided by 72,134,000 gives 6.914 lbs. traction.

A table calculated by this rule is given. A diminished area of contact sometimes increases traction, and a non-uniform distribution of lines may occasion departures from it. The above rule and the table alluded to are practically only accurate for uniform conditions. The reciprocal of the rule is applied in determining the lines of force of a magnet experimentally.

Rule 66. The lines of force which can pass through a magnet core with economy are determined by the tables, keeping in mind that it is not advisable to let the permeability fall below 200–300. From them a number is taken (40,000 lines per square inch for cast iron or

100,000 lines per square inch for wrought iron are good general averages) and is multiplied by the cross-sectional area of the magnet core.

Rule 67. To calculate the magnetizing force in ampere turns required to force a given number of magnetic lines through a given permeance, multiply the desired lines of force by the reluctance determined as below.

Rule 68. *a.* **The reluctance of a core or of any portion thereof for inch measurements is equal to the product of the length of the core or of the portion thereof by 0.3132 divided by the product of its cross-sectional area and permeability.**

b. **The reluctance for centimeter measurements is equal to the length of the core divided by the product of 1.2566, by the cross-sectional area and the permeability.**

EXAMPLES.

440,000 lines are to be forced through a bar of wrought iron 10 inches long and 4 square inches in area; calculate its reluctance and the magnetizing force in ampere turns required to effect this magnetization.

Solution: The reluctance (*a*) = 10 × .3132 ÷ (4 × permeability). 440,000 lines through 4 square inches area is equal to 110,000 lines through 1 square inch; for this intensity and for wrought iron the permeability = 166. 166 × 4 = 664. The reluctance therefore = 3.132 ÷ 664 = .0047. The magnetizing force in ampere turns = 440,000 × .0047 = 2068.

The same number of lines are to be forced through a bar 25.80 square centimeters area and

25.40 centimeters long. Calculate the ampere turns.

Solution: 440,000 lines through 25.80 sq. cent. $= \frac{440,000}{25.80} = 17,054$ through 1 sq. cent., for which the permeability = 161. The reluctance therefore, (*b*) $= 25.40 \div (1.2566 \times 25.80 \times 161) = .0048$. The ampere turns $= 440,000 \times .0048 = 2112$.

Magnetic Circuit Calculations.

Practically useful calculations include always the attributes of a full magnetic circuit, because magnetization can no more exist without a circuit than can an electric current. In practice an electromagnetic circuit consists of four parts: 1, The magnet cores; 2 and 3, the gaps between armature and magnet ends; 4, the armature core. To calculate the relations of magnetizing force to magnetization the sum of the reluctances of these four parts has to be found. A further complication is introduced by leakage. The permeability of well magnetized iron being so low, not exceeding 150 to 300 times that of air, a quantity of lines leak across through the air from magnet limb to magnet limb. Leakage is included in the sum of the reluctances by multiplying the reluctance of the magnet core by the *coefficient of leakage,* which is calculated for each case by more or less complicated methods. For parallel cylindrical limb magnets the calculation is

exceedingly simple. The calculation in all cases is simplified by the fact already stated, that in air permeability is always equal to unity, whatever the degree of magnetization. For copper and other non-magnetizable metals the variation from unity is so slight that it may, for practical calculations, be treated as unity.

LEAKAGE OF LINES OF FORCE.

Leakage is the magnetic flux through air from surfaces at unequal magnetic potential, such as north and south poles of magnets. It is measured by lines of force and is proportional to the relative permeance of its path.

The coefficient of leakage of a magnetic circuit is the quotient obtained by dividing the total magnetic flux by the flux through the armature. The total magnetic flux is the maximum flux through the magnet core.

Rule 69. To obtain the coefficient of leakage divide the permeance of the armature core and of the two gaps plus one-half the permeance of air between magnet limbs by the permeance of the armatüre core and of the two gaps.

EXAMPLE.

The total flux through an armature core is found to be at the rate of 70,000 lines per square inch, and the armature core is 3 inches diameter and 10 inches long. The average length of travel of the magnetic

lines through it is $1\frac{1}{4}$ inches. The air gaps are 10 × 3 inches area and $\frac{1}{4}$ inch thick. The permeance between the limbs of the magnet is 500. Calculate the coefficient of leakage.

Solution: 70,000 lines per square inch gives a permeability of 1,921. By Rule 68 the reluctance of the armature core is $\frac{1\frac{1}{4}}{30 \times 1921} \times .3132 = .000008$. The reluctance of a single air gap is $\frac{\frac{1}{2}}{30} \times .3132 = .0052$. Thus the armature reluctance is so small that it may be neglected. The permeance of the two air gaps is given by $\frac{1}{.0052 \times 2} = 100$ (about). The coefficient of leakage $= \frac{100 + 250}{100} = 3.5$.

As the coefficient of leakage is the factor used in these calculations, the permeance of the leakage paths is the desired factor for its determination. In the case of cylindrical magnet cores parallel to each other, they are obtained from a special table given in its place later. It is thus calculated and used. The least distance separating the cores (b) is divided by the circumference of a core (p) giving the ratio ($\frac{b}{p}$) of least distance apart to perimeter of a core. The number corresponding in columns 3 or 5 is multiplied by the length of a core. The product is the permeance. Columns 2 and 3 give the reluctance. To reduce to average difference of magnetic potential divide by 2.

EXAMPLE.

Calculate the permeance between the legs of a magnet, 3 inches in diameter and 12 inches high and 5 inches apart.

Solution: The perimeter $= 3 \times 3.14 = 9.42$. $\frac{5}{9.42} = \frac{1}{2}$ or .5 nearly. From the table of permeability we find 6.278. Multiplying this by 12 we have $6.278 \times 12 = 75.336$, the permeance. Dividing by 2 we have $\frac{75.336}{2} = 37.668$, the permeance for use in the calculation of leakage coefficient.

It will be observed that this calculation is based entirely on the ratio stated, and that absolute dimensions have no effect on it.

For flat surfaces, parallel and facing each other, the following method precisely comparable to the rule for specific resistance is used:

Rule 70. The permeance of the air space between flat parallel surfaces is equal to their average area multiplied by 3.193 and divided by their distance apart, all in inch measurements.

EXAMPLE.

Determine the permeance between the two facing sides of a square cored magnet 15 inches long, 3 inches wide and 8 inches apart.

Solution: $3 \times 15 = 45$ (the average area); $45 \times 3.193 \div 8 = 17.96$. For use in calculations it should be divided by 2 giving 8.98. This division by 2 is to reduce it to the average difference of magnetic potential between the two magnet legs.

CALCULATIONS FOR MAGNETIC CIRCUITS.

A magnetic circuit is treated like an electric one. The permeance (analogue of conductance) or reluctance (analogue of resistance) is calculated for its four parts, magnet core, two air gaps, and armature core. The leakage coefficient is determined and applied. The requisite magnetizing force is calculated in the form of ampere turns (the analogue of volts of E. M. F.). The preceding leakage rules cover the case of parallel leg magnets. For others a slight change is requisite in the leakage calculations, but in practice an average can generally be estimated.

EXAMPLE.

Assume the magnet and armature of a dynamo. The magnet is of cast iron, each leg is cylindrical in shape, 4 inches in diameter and 20 inches high. From center to center of leg the distance is 9 inches. The armature core of soft wrought iron is 4 inches in diameter and 8 inches long, the pole pieces curving around it are 4 inches, measured on the curve inside, by 8 inches long. The air gap is ¼ inch thick. Calculate the reluctance of the circuit and the ampere turns for 500,000 lines of force.

Solution: The pole pieces approach within 2¼ inches of each other. This leaves 1¾ inches of the diameter of the armature core embedded or included within or embraced by them. One-half of this

amount may be taken and added to $2\frac{1}{2}$ giving $3\frac{1}{4}$ as the average depth of core for an area $4 \times 8 = 32$ square inches. The lines per square inch of armature core are $\frac{500,000}{32} = 15,625$ lines per square inch. By the table of permeability, 4650 is given for permeability for 30,000 lines in soft iron. For 15,625 lines per square inch 9,000 can safely be taken for permeability. Its relative reluctance is therefore $\frac{3\frac{1}{4}}{32 \times 9000} = .000011$ relative armature core reluctance. (1)

The relative reluctance of one air gap (permeability = 1) is $\frac{1}{4} \div 32 = .0078$ and $.0078 \times 2 = .0156 =$ air gaps reluctance (2).

The ratio $\frac{h}{p}$ of the table for determining the leakage between cylindrical magnet legs is $\frac{5}{4 \times 3.14} = .4$. 5 is the distance between the legs. Permeance corresponding thereto is 6.897, which multiplied by 20, the length of the legs, and divided by 2 for average magnetic potential difference gives 68.97 for relative effective permeance (3).

The relative reluctance of the air gaps and armature core is .015611; the reciprocal or permeance is 64.06 (4).

For coefficient of leakage we have $(64.06 + 68.97) \div 64.06 = 2.08$ (5).

To find the relative reluctance of the magnet core whose yoke may be taken as of mean length 9 inches

and of area equal to that of the core ($3.14 \times 2^2 = 12.56$) we have to first determine the permeability. $\frac{500,000}{12.56} = 40,000$ lines per square inch, corresponding to a permeability of 258. For the effective reluctance of the magnet core introducing the factor of leakage (2.08) we have the expression $\frac{(20 + 20 + 9) \times 2.08}{12.56 \times 258} = .0314$.

To get ampere turns, we add the reluctances of circuit, multiply by .3132 and by the required lines, $(.000011 + .0156 + .0314) \times .3132 \times 500,000 =$ 7362 ampere turns required.

In the above calculations, the multiplication by .3132 was omitted to save trouble, relative reluctances only being calculated, until the end when one multiplication by .3132 brought out the ampere turns. The leakage appears excessive partly because of the high reluctance of the two air gaps. These should be increased in area and reduced in depth if possible. The leakage is also high on account of the legs of the magnet being close together. Were these separated, a larger armature core might be used, justifying a lower speed or rotation of armature, reducing reluctance of air gaps by increasing their area, and reducing leakage between magnet legs by increasing their distance. The magnet legs might also be made shorter, thus reducing leakage.

Thus assume the magnet core of the same cross-sectional area, but only 10 inches long and with a distance apart of legs of 7 inches, giving a 7×10 inch armature core and pole piece areas (air gap areas) of $7 \times 10 = 70$ sq. inches.

For leakage ratio we have $(\frac{b}{p}) = \frac{7}{12.56} = .56$ giving from the proper table 6.000 (about), $\frac{6.000 \times 10}{2} = 30$ relative permeance of air space between legs.

For air gaps reluctance $\frac{1}{4} \div 70 = .00357$ which for the two gaps gives .00714 relative reluctance.

Treating the armature core as a prism $7 \times 10 = 70$ sq. inches area and 5 inches altitude, we have for lines per sq. inch $500{,}000 \div 70 = 7000$ giving it about 9000 and reluctance as $5 \div (70 \times 9{,}000) = .000008$ reluctance.

Air gaps and armature core reluctance $= .007148$ and permeance $= \frac{1}{.007148} = 139$.

Coefficient of leakage $= \frac{139 + 30}{139} = 1.21$.

If the depth of the air gaps was reduced to $\frac{1}{8}$ inch the coefficient of leakage would then be about 1.11.

Every surface in a magnet leaks to other surfaces and the leakage from leg to leg is sometimes but one third of the total leakage. In practice the total leakage often runs as high as 50%, giving a coefficient of 2.00 and in other cases as low as 25%, giving a coefficient of 1.33.

DYNAMO ARMATURES.

An armature of a dynamo generally comprises two parts—the core and the winding. The core is of soft iron. Its object is to direct and concentrate the lines of force, so that as many as possible of them shall be cut by the revolving turns or convolutions of wire. The winding is usually of wire. It is sometimes, however, made of ribbon or bars of copper. Iron winding has also been tried, but has never obtained in practice. The object of the winding is to cut the lines of force, thereby generating electro motive force. The number of the lines of force thus cut in each revolution of the armature is determined from the intensity of the field per unit area, and from the position, area and shape of the armature, coils and pole pieces. The number thus determined, multiplied by the number of times a wire cuts them in a second, and by the effective number of such wires, gives the basis for determining the voltage of the armature.

Rule 71. One volt E. M. F. is generated by the cutting of 10^8 (100,000,000) lines of force in one second.

EXAMPLES.

A single convolution of wire is bent into the form of a rectangle 7 × 14 inches. It revolves 25 times a second in a field of 20,000 lines per square inch. What E. M. F. will it develop at its terminals?

Solution: The area of the rectangle is 7 × 14 = 98

square inches. Multiplying this by the lines of force in a square inch, we have $98 \times 20{,}000 = 1{,}960{,}000$. Each side of the rectangle cuts these lines twice in a revolution, and makes 25 revolutions in a second. This gives $25 \times 2 \times 1{,}960{,}000 = 98{,}000{,}000$ lines cut per second, corresponding to $98 \times 10^6 \times 10^{-8} = 98 \times 10^{-2} = \frac{98}{100}$ volts E. M. F. generated, or $\frac{98000000}{100000000} = \frac{98}{100}$ volts.

The field of the earth in the line of the magnetic dip = .5 line per square centimeter. Calculate a size, number of layers, and speed of rotation for a one volt earth coil.

Solution : We deduce from the rule the following: Area of coil $\times$ revolutions per sec. $\times$ convolutions of wire $\times .5 \times 10^{-8} = .5$. We may start with revolutions per second, taking them at 20. Next we may take 50,000 convolutions. $20 \times 50{,}000 \times .5 = 500{,}000$. This must be multiplied by 200 to give 10^8; in other words, the average area within the wire coils must be 100 square centimeters, or 10×10 centimeters. $2 \times 100 \times 2000 \times 500 \times .5 = 10^8$, and $10^8 \times 10^{-8} = 1$ volt.

Rule 72. The capacity of an armature for current is determined by the cross-section of its conductors. This should be such as to allow 520 square mils per ampere = 1923 amperes per square inch area.

EXAMPLE.

A drum armature coil is of 4 inches diameter, and is wound with wire $\frac{1}{300}$ of the periphery of the

drum in diameter; the wire is 100 feet long. Its E. M. F. is 90 volts. What is the lowest admissible external resistance?

Solution: The circumference of the drum is 3.14 × 4 = 12.56 inches. The diameter of the wire is $\frac{12.56}{300}$ = .0418 in. or 42 mils. The area of the wire is $21^2 \times 3.14$ = 1387 square mils. By the rule the allowable current in amperes for a single lead of such wire is $\frac{1387}{520}$ = 2.66 amperes. But on a drum armature the wire lies with two leads in parallel. Hence it has double the above capacity or 2.66 × 2 = 5.32 amperes. The resistance of such wire may be taken at .137 ohm. By Ohm's law the total resistance for the current named must be $\frac{90}{5.32}$ or 17 ohms. The external resistance is given by 17 — .137 = 16.863 ohms.

These two rules enable us to calculate the capacity of any given armature. Certain constants depending on the type of armature have to be introduced in many cases.

Drum Type Closed Circuit Armatures.

For these armatures the following rules of variation hold, when they do not differ too much in size, and are of identical proportions.

Rule 73. *a.* **The E. M. F. varies directly with the square of the size of core and with the number of turns of wire.**

b. **The current capacity varies with the sixth power of the size of core for identical E. M. F.**

c. **The resistance varies directly with the cube of the number of turns and inversely with the size of core.**

d. **The amperage on short circuit varies directly with the cube of the size and inversely with the square of the number of turns.**

In these rules the proportions of the drum are supposed to remain unchanged. Size may be referred to any fixed factor such as diameter, as lineal size is referred to.

These rules enable us to calculate an armature for any capacity and voltage. As a starting point a given intensity of field, speed of rotation, and number of turns of wire and size of wire has to be taken. The wire is selected to completely fill the periphery of the drum. Then a trial armature is calculated of the required voltage and its amperage is calculated. With this as a basis, by applying Rule 73, sections *a* and *b*, the size of an armature for the desired current capacity is calculated, the E. M. F. being kept identical. As a standard for medium sized machines 20% of the turns of wire may be considered inactive.

EXAMPLE.

Calculate a 100 volt, 20 ampere armature, whose length shall be twice its diameter, to work at a speed of 15 revolutions per second.

Solution: Take as intensity of field 20,000 lines per square inch. Allow 80% of active turns of

wire. Start with a core $8 \times 16 = 128$ square inches, including $128 \times 20,000 = 256 \times 10^4$ lines of force. The given speed is 15 rotations per second. For the number of active turns of wire per volt we have to divide 10^8 or 100,000,000 by one half the lines of force cut by one wire per second. This number is $256 \times 10^4 \times 15$, or 38,400,000; and $\frac{100,000,000}{38,400,000} = 2.6$ turns. For 100 volts, therefore, 260 active turns are needed. If one half the lines were not taken the result would be one half as great, because each line cuts each line of force twice in a revolution, and in the computation a single cutting per revolution only is allowed for.

The reason for thus taking one half the lines cut by a single wire as a base is because in the drum armature the wires work in two parallel series, giving one half the possible voltage. The actual turns are $260 \div .80 = 325$, say 324 turns. Assume it to be laid in two layers giving 162 turns to the layer. The space occupied by a wire is equal to the perimeter divided by the number of wires or $\frac{25}{162} = .154$ in. Allowing 25% for thickness of insulation, lost space, etc., we have .115 in. or 115 mils as the diameter of the wire. In the drum armature as just stated the wire is parallel, so that the area of one lead of wire has to be doubled, giving $10,573 \times 2$ square mils as the area of the two parallel leads. This is enough for 40 amperes or double the amperage required. This capacity is reached by taking

520 square mils per ampere as the proper cross-sectional area of the wire. (Rule 72.)

We must therefore reduce the size to give $\frac{1}{2}$ the ampere capacity; this reduction (by Rule 73 *b*) is in the ratio $1 : \frac{1}{2}^{\frac{1}{6}} = 1 : .89018$; the size therefore is $8 \times .89018$ diameter by $16 \times .89018$ length $= 7.12 \times 14.24$ inches.

Applying *a* for voltage we have for the same number of turns on the new armature a voltage in the ratio of $1 : .89018^2$ or about $\frac{8}{10}$ of that required. We must therefore divide the number of turns in the trial armature by $.89018^2$, giving for the number of turns $\frac{324}{.7924} = 409$, say 410 turns.

To prove the operation we first determine the voltage of the new armature. Its area is $7.12 \times 14.24 = 101.4$ square inches including 2,028,000 lines of force. The active wires are $410 \times .8 = 328$. We have for the voltage $= \frac{4,056,000 \times 164 \times 15}{10^8} = 99.78$ volts.

The relative capacity of the wire is deduced from the square of its diameter. The circumference of the new armature is $7.12 \times 3.14 = 22.3568$. There are 205 turns in a layer giving as diameter of wire $\frac{22.3568}{205} = .1091$ mils. This must be squared, giving .0119, and compared with the square of the corresponding number for the original armature. This number was $25 \div .162 = .154$ inch. $.154^2 = .02371$

and .01190 + .02371 = $\frac{1}{2}$ (nearly), showing that the new armature has one half the ampere capacity of the old, or 40 × $\frac{1}{2}$ = 20 amperes as required.

The gauge of the wire is reached by making the same allowance for insulation and lost space, viz., 25%. .1091 × .75 = .0818 in. or 81.8 mils diameter, for size of wire. Of course there is nothing absolute about 25% as a loss coefficient; it will vary with style of insulation and even to some extent with the gauge of wire. But as Rule 73 is based upon the assumption that this loss is a constant proportion of the diameter of the wire, too great a variation of sizes should not be allowed in its application. In other words the trial armature should be as near as possible in size to the final one.

Suppose on the other hand that an armature for 100 amperes was required. This is for 2½ times 40 amperes (the capacity of the first calculated or trial armature).

Applying *b* we extract the 6th root of 2½. $(2\frac{1}{2})^{\frac{1}{6}}$ = 1.1653 (by logarithms or by a table of 6th roots). The size of the new armature is therefore 8 × 1.1653 by 16 × 1.1653 or 9.3224 × 18.6448 inches.

Applying *a* for voltage we have for the same number of turns of the new armature a voltage in the ratio of 1.1653^2 : 1 or 1.358 times too great. We

must therefore multiply the original turns by the reciprocal of 1.358, giving $\frac{324}{1.358} = 239$ turns.

To prove the voltage, we multiply 239 by .8 for the active turns of wire, giving 191.2 turns. The area of the armature is $9.32 \times 18.64 = 172.7$ square inches. For voltage this gives $172.7 \times 191.2 \times 20{,}000 \times 15 \times 10^{-8} = 98.6$ volts (about).

To prove the capacity we must divide the circumference of the new armature, $9.32 \times 3.14 = 29.26$ inches, by the turns of wire in one layer, $\frac{239}{2} = 120$ turns (about). This gives a diameter of 244 mils (nearly). The ratio of capacities of the original and this wire is $.244^2 \div .154^2$ inches $= .059536 \div .02371 = 2.51$ corresponding to $40 \times 2.51 = 100$ amperes.

These results, owing to omissions of decimals, do not come out exactly right and it is quite unnecessary that they should. The accuracy is ample for all practical purposes. For armature dimensions it would be quite unnecessary to work out to the second decimal place. It would answer to take as armature sizes in the two cases given $7 \times 14\frac{1}{2}$ inches and $9\frac{1}{4} \times 18\frac{3}{4}$ inches.

It is also to be noted that a very low rate of rotation was taken. 25 to 30 turns per second would not have been too much. The latter would give double the voltage and the same amperage.

Field Magnets of Dynamos.

The calculation for a magnetic circuit given on

pages 92 et seq., is intended to supply an example of the calculation of the circuit formed by a field magnet and its armature, such as required for dynamos. The leakage of lines of force is and can only be so incompletely calculated that it is probably the best and most practical plan to assume a fair leakage ratio and to make the magnet cores larger than required by the lines of force of the armature in this ratio. A low multiplier to adopt is 1.25, which is lower than obtains in most cases; 1.50 is probably a good average.

Rule 74. The cross-sectional area of the field-magnet cores is equal to the lines of force in the field divided by the magnetic flux (column B) for the material selected and corresponding to the chosen permeability (μ), multiplied by the leakage coefficient.

A good range for permeability is from 200 to 400 giving for wrought iron from 100,000 to 110,000 lines of force per square inch and for cast iron from 35,000 to 45,000 lines per square inch; for the field from 15,000 to 20,000 lines per square inch may be taken.

The permeability table gives data for different qualities of iron.

EXAMPLE.

Taking the 100 volt 100 ampere armature last calculated, determine the size of field-magnet cores to go with it, and the ampere turns and other data.

Solution: Assume 20,000 lines of force per square

inch in the field, 45,000 in cast iron and 110,000 in wrought iron core and a leakage coefficient of 1.25. We have for total lines of force passing through armature $172.7 \times 20{,}000 = 3{,}454{,}000$; cross-sectional area for cast iron core $\frac{3{,}454{,}000}{45{,}000} \times 1.25 = 96$ square inches; cross-sectional area for wrought iron core $\frac{3{,}454{,}000}{110{,}000} \times 1.25 = 39$ square inches.

As length of cores we may take 20 inches with a distance between them of 10 inches. Assume wrought iron to be selected. If cylindrical they would be 7 inches in diameter to give the required cross-sectional area. The yoke connecting them would average in length $10 + 7 = 17$ inches, giving for magnet cores and yoke a length of $17 + 20 + 20 = 57$ inches. The reluctance of cores and yoke (Rule 68) $= \frac{57 \times .3132}{72 \times 200}$ (taking $\mu = 200$) which reduces to .00132 (1).

The armature area is 172.7 inches. As average length of the path of lines of force through it 5 inches may be taken. As it passes only 20,000 lines of force per square inch of field its permeability is high, say 9000. Its reluctance is given by $\frac{5 \times .3132}{173 \times 9000}$. This is so low that it may be neglected.

The area of each air-gap may be taken as 173 square inches, and of depth of two windings *plus* about $\frac{1}{10}$ inch for clearance or windage giving $(.224 \times 2) + .1 =$ about .6 inch for its depth. Its

reluctance is $\frac{.6 \times .3132}{173} = .00108$. As there are two air gaps we may at once add their reluctances giving .00216 (2).

By Rule 67 the ampere turns are equal to the product of the reluctances (1) and (2), by the lines of force giving $(.00121 + .00216) \times (172.7 + 20,000) = 11640$ ampere turns.

The proper size of wire for series winding may be determined by Sir William Thompson's rule that in series wound dynamos the resistance of the field magnet windings should be $\frac{2}{3}$ that of the armature. The length of the wire in the armature is equal approximately, to the circumference $9.32 \times 3.14 = 29.26$ multiplied by the number of turns (240) giving $29.26 \times 240 = 7022$ inches.

The wire turns on the field magnets are found by dividing the ampere turns by the amperes giving $\frac{11632}{50} = 232$ turns. The circumference of the magnet leg is $7.0 \times 3.14 = 22$ inches. The total length of wire is therefore, approximately, $232 \times 22 = 5104$ inches.

To compare the resistances we must use $\frac{7022}{4}$ for the length of the armature wire, because it is in parallel, and therefore is $\frac{1}{2}$ the length and $\frac{1}{4}$ the resistance of the full wire in one length. Dividing by 4 introduces this factor.

As the resistances of the wires are to be in the ratio of 2 : 3, we have by Rule 13 (calling the thickness of armature wire $244 \times .75 = 183$ mils to allow for

insulation, etc.), $2 : 3 :: 183^2 \times 5104 : x^2 \times \frac{7022}{4}$, and solving we find $x^2 = 73026 \therefore x = 270$ mils.

For shunt winding Sir William Thompson's rule is that the product of armature and field resistance should equal the square of the external resistance. The latter may be taken (Ohm's law) as equal to $\frac{100 \text{ volts}}{100 \text{ amperes}} = 1$ ohm. Properly the armature resistance should be allowed for, but it is so small that it need not be included. We have therefore, armature resistance $\times$ field resistance $= 1^2 \times 1$.

The armature resistance is .0419 ohms. Therefore the field resistance is $\frac{1}{.0419} = 24$ ohms. The current through this is equal to $\frac{100}{24} = 4$ amperes (nearly). Therefore $\frac{11639}{4} = 2910$ turns of wire are needed. The length of such wire will be $\frac{22 \times 2910}{12}$ $= 5335$ feet. The resistance is about 4.4 ohms per 1000 feet corresponding to about .48 mils diameter.

The Kapp Line.

Mr. Gisbert Kapp, C. E. who has given much investigation to the problems of the magnetic circuit and especially to dynamo construction, is the originator of this unit. He considered the regular C. G. S. line of force to be inconveniently small. He adopted as a line of force the equivalent of 6000 C. G. S. lines and as the unit of area one square inch. Therefore to reduce Kapp lines to regular lines of force they must be multiplied by 6000, and ordi-

nary lines of force must be divided by 6000 to obtain Kapp lines. These lines are often used by English engineers. The regular system is preferable and by notation by powers of ten can be easily used in all cases.

CHAPTER X.

DEMONSTRATION OF RULES.

In the following chapter we give the demonstration of some of the rules. As this is not within the more practical portion of the work, algebra is used in some of the calculations. It is believed that rules not included in this chapter, if not based on experiment, are such as to require no demonstration here.

Rule 1 to 6, pages 13 and 14. Ohm's law was determined experimentally, and all the six forms given are derived by algebraic transposition from the first form which is the one most generally expressed.

Rule 8, page 19. This is simply the expression of Ohm's law as given in Rule 1, because in the case of divided circuits branching from and uniting again at common points, it is obvious that the difference of potential is the same for all. Hence the ratio as stated must hold.

Rule 9, page 20. This rule is deduced from Rule 8. It first expresses by fractions the relations of the current. Next these fractions are reduced to a common denominator, so as to stand to each other in

the ratio of their numerators. By applying the new common denominator made up of the sum of the numerators the ratio of the numerators is unchanged, and the ratio of the new fractions is the same as that of their numerators, while by this operation the sum of the new fractions is made equal to unity. Thus by multiplying the total current by the respective fractions it is divided in the ratio of their numerators, which are in the inverse ratio of the resistances of the branches of the circuit and as the sum of the fractions is unity, the sum of the fractions of the current thus deduced is equal to the original current.

Rule 10, page 21. Resistance is the reciprocal of conductance. By expressing the sum of the reciprocals of the resistances of parallel circuits we express the conductance of all together. The reciprocal of this conductance gives the united resistance.

Rule 11, page 22. This is a form of Rule 10. Call the two resistances x and y. The sum of their reciprocals is $\frac{1}{x}+\frac{1}{y}$ which is the conductance of the two parallel circuits or parts of circuits. Reducing them to a common denominator we have: $\frac{y}{xy}+\frac{x}{xy}$ which equals $\frac{x+y}{xy}$, whose reciprocal is $\frac{xy}{x+y}$.

Rule 17, page 31. Taking the diameter of a wire as d, its cross sectional area is $\frac{\pi d^2}{4}$. The resistance is inversely proportional to this or varies directly with $\frac{4}{\pi d^2}=\frac{1.2737}{d^2}$. As the resistance of a conductor

varies also with its length and specific resistance we have as the expression for resistance:

$$\frac{\text{Sp. Res.} \times 1.2737 \times 1}{d^2}$$

Rule 18, page 32. Assume two wires whose lengths are l and l_1, their cross sectional areas a and a_1, their specific resistances s and s_1, and their resistances r and r_1. From preceding rules we have for each wire: $r = s\,\frac{l}{a}$ (1) and $r_1 = s_1\,\frac{l_1}{a_1}$ (2).

Dividing (1) by (2) we have:

$$\frac{r}{r_1} = \frac{s}{s_1} \times \frac{l}{l_1} \times \frac{a_1}{a} \text{ (3).} \qquad \text{(Day)}$$

If we take the reciprocal of either member of this equation and multiply the other member thereby it will reduce it to unity, or:

$$\frac{r_1}{r} \times \frac{s}{s_1} \times \frac{l}{l_1} \times \frac{a_1}{a} = 1$$

For convenience this is put into a shape adapted for cancellation.

Rule 20, page 38. This is merely the expression of Ohm's Law, Rule 3.

Rule 22, page 40. Call the drop e, the combined resistance of the lamps R, and the resistance of the leads x. Then as the whole resistance is expressed as 100 (because the work is by percentage) the difference of potential for the lamps is 100—e. By Ohm's law we have the proportion: $100-e : e :: R : x$ or

$$x = \frac{e\,R}{100-e}$$

Rule 25, page 44. From Rule 22 we have:

$$x = \frac{e\,R}{100-e} \qquad (1)$$

Call the resistance of a single lamp r, then we have by Rule 12:

$$R = \frac{r}{n} \qquad (2)$$

Substituting this value of R in equation (1) we have:

$$x = \frac{e\,r}{n \times (100-e)} \qquad (3)$$

From Rule 24 we have, calling the cross-section a:

$$a = \frac{l \times 10.79}{x} \qquad (4)$$

Substituting for x its value from equation (3) we have:

$$a = \frac{l \times 10.79 \times n \times (100-e)}{e\,r} \qquad (5)$$

But as l expresses the length of a pair of leads, not the total length of lead but only one-half the total, the area should be twice as great. This is effected by using the constant $10.79 \times 2 = 21.58$ in the equation giving:

$$a = \frac{l \times 21.58 \times n \times (100-e)}{e\,r}$$

Rule 28, page 48. Assuming the converter to work with 100% efficiency (which is never the case), the watts in the primary and secondary must be equal to each other or:

$$C^2\,R = C_1{}^2\,R_1, \text{ and } R = \frac{C_1{}^2}{C^2}\,R_1,$$

or the resistances of primary and secondary are in the ratio of the squares of the currents. The direct ratio is expressed by the ratio of conversion, when squared it gives the ratio of the squares as required.

Rule 37, page 59. Let d = diameter of the wire in centimeters. The resistance of one centimeter of such a wire in ohms = Sp. Resist. $\times 10^{-6} \times \frac{4}{\pi d^2}$. The specific resistance is here assumed to be taken in microhms. The quantity of heat in joules developed in such a wire in one second is equal to the square of the current in C. G. S. units, multiplied by the resistance in C. G. S. units and divided by 4.16×10^7, the latter division effecting the reduction to joules. 1 ohm = 10^9 C. G. S. units of resistance. Multiplying the expression for ohmic resistance by 10^9 we have: Sp. Resist. $\times 10^3 \times \frac{4}{\pi d^2}$. 1 ampere = 10^{-1} C. G. S. unit. If we express the current in amperes we must multiply it by 10^{-1}, in other words take one-tenth of it. Our expression then becomes for heat developed in one second

$$\left(\frac{c}{10}\right)^2 \times \frac{\text{Sp. Resist.} \times 10^3 \times 4}{\pi\, d^2 \times 4.16 \times 10^7}$$

The area of one centimeter of the wire is $\pi\, d$ square centimeters. The heat developed per square centimeter is found by dividing the above expression by $\pi\, d$ giving:

$$\left(\frac{c}{10}\right)^2 \times \frac{\text{Sp. Resist.} \times 10^3 \times 4}{\pi^2\, d^3 \times 4.16 \times 10^7}$$

The heat developed is opposed by the heat lost which we take as equal to $\frac{1}{4000}$ per square centimeter per degree Cent. of excess above surrounding medium. Therefore taking t° as the given tempera-

ture cent. we may equate the loss with the gain thus:

$$\frac{t^\circ}{4000} = \left(\frac{c}{10}\right)^2 \times \frac{\text{Sp Resist.} \times 10^3 \times 4}{\pi^2\, d^3 \times 4.16 \times 10^7}$$

$$d^3 = \frac{c^2 \times \text{Sp. Resist.} \times 10^3 \times 4 \times 40000}{\pi^2 \times 4.16 \times 10^7 \times t^\circ} =$$

$$\frac{c^2 \times \text{Sp. Resist.} \times .00039}{t^\circ}$$

Rule 51, page 69. Call the external resistance r; number of cells n; resistance of one cell R; E. M. F. of one cell E; E. M. F. of outer circuit e.

Then from Ohm's law we have:

$$C = \frac{n\,E}{n\,R + R} \qquad (1)$$

which reduces to:

$$n = \frac{C\,r}{E - C\,R} \qquad (2)$$

but $C\,r = e$.

$$\therefore\ n = \frac{e}{E - C\,R} \qquad (3)$$

Rule 54, page 72. This rule is deduced from the following considerations. The current being constant the work expended in the battery and external circuit respectively will be in proportion to their differences of potential or E. M. F's. But these are proportional to the resistances. Therefore the resistance of the external circuit r should be to the resistance of the battery R as efficiency: 1 — efficiency or r : R :: efficiency : 1 — efficiency or $R = \frac{(1 - \text{efficiency}) \times r}{\text{efficiency}}$. The rest of the rule is deduced from Ohm's law.

Rule 57, page 74. This rule gives the nearest approximation attainable without irregular arrangement of cells. By placing some cells in single series and others two or more in parallel, an almost exact arrangement for any desired efficiency can be obtained. Such arrangement are so unusual that it is not worth while to deduce any special rule for them. Thus taking the example given on page 74 the impossible arrangement of 1.4 cells in parallel and 63 in series would give the desired current and efficiency. The same result can be obtained by taking 72 cells in 36 pairs with a resistance of $36 \times \frac{1}{12} = 3$ ohms, and adding to them 27 cells in series with a resistance of $27 \times \frac{1}{6} = 4\frac{1}{2}$ ohms, a total of $7\frac{1}{2}$ ohms. The E. M. F. is equal to $(36 + 27) \times 2 = 126$ volts. The total cells are $72 + 27 = 99$.

Rule 58, page 76. One coulomb of electricity liberates from an electrolyte .000010384 gram of hydrogen. This has been determined experimentally. Let H be the heat liberated by the chemical combining weight of any body combining with another. H is taken in kilogram calories. Hence it follows that for a quantity of the substance equal to .000010384 gram × chemical combining weight, the heat liberated will be equal to H × .000010384, which corresponds to a number of kilogram meters of work expressed by .000010384 × H × 424. The work done by a current in kilogram-meters =

$\frac{\text{volts} \times \text{coulombs}}{9.81}$ or for one coulomb $= \frac{\text{volts}}{9.81}$. This expresses the work done by one coulomb. Let the volts $= E$, and equate these two expressions:

$\frac{E}{9.81} = .000010384 \times H \times 424,$

which reduces to

$$E = H \times .043.$$

Rule 61, page 78. For the work (in kilogram-meters) done by a current (volt-coulombs) we have the general expression:

$$W = \frac{\text{volts} \times \text{coulombs}}{9.18} \text{ or } \frac{E\,Q}{9.81} \qquad (1)$$

Making $W = 1$ (i. e. one kilogram-meter) and transforming, we have, as the coulombs corresponding to 1 kilogram-meter:

$$Q = \frac{9.81}{E} \qquad (2)$$

One coulomb of electricity liberates a weight (in grams) of an element equal to the product of the following: .000010384 $\times$ equivalent of element in question $\times$ number of equivalents $\div$ valency of the element. Therefore, the coulombs corresponding to one kilogram-meter, liberates this weight multiplied by $\frac{9.81}{E}$ or indicating weight by G.

$$G = \frac{.000010384 \times \text{equiv.} \times \text{number equiv.}}{\text{valency}} \times \frac{9.81}{E} \qquad (3)$$

but $.000010384 \times 9.81 = .000101867.$

$$\therefore G = \frac{\text{equiv.} \times n \times .000101867}{E \times \text{valency}} \qquad (4)$$

Rule 73, page 98-99. The voltage of an armature of

a definite number of turns of wire and a fixed speed, varies with the lines included within its longitudinal area, as such lines are cut in every revolution. These lines vary with its area, and the latter varies with the square of its linear dimensions.

To maintain a constant voltage if the size is changed, the number of turns must be varied inversely as the square of the linear dimensions. This ensures the cutting of the same number of lines of force per revolution.

If, therefore, its size is reduced from x to $\frac{1}{x}$ the turns of wire must be changed from x to x^2. The relative diameters of the two sizes of wire is found by dividing a similar linear dimension by the relative size of the wire. But $\frac{1}{x} \div x^2 = \frac{1}{x^3}$ = diameter of the wire for maintenance of a constant voltage with change of size.

The capacity of a wire varies with the square of its diameter and $(\frac{1}{x^3})^2 = \frac{1}{x^6}$.

Therefore the amperage, if a constant voltage is maintained, will vary inversely as the sixth power of the linear dimensions of an armature.

CHAPTER XI.

NOTATION IN POWERS OF TEN.

THIS adjunct to calculations has become almost indispensable in working with units of the C. G. S. system. It consists in using some power of 10 as a multiplier which may be called the factor. The number multiplied may be called the characteristic. The following are the general principles.

The power of 10 is shown by an exponent which indicates the number of ciphers in the multiplier. Thus 10^2 indicates 100; 10^3 indicates 1000 and so on.

The exponent, if positive, denotes an integral number, as shown in the preceding paragraph. The exponent, if negative, denotes the reciprocal of the indicated power of 10. Thus 10^{-2} indicates $\frac{1}{100}$; 10^{-3} indicates $\frac{1}{1000}$ and so on.

The compound numbers based on these are reduced by multiplication or division to simple expressions. Thus: $3.14 \times 10^7 = 3.14 \times 10{,}000{,}000 = 31{,}400{,}000$. $3.14 \times 10^{-7} = \frac{3.14}{10{,}000{,}000}$ or $\frac{314}{1{,}000{,}000{,}000}$. Regard must be paid to the decimal point as is done here.

To add two or more expressions in this notation if the exponents of the factors are alike in all respects, add the characteristics and preserve the same factor. Thus:

$$(51 \times 10^6) + (54 \times 10^6) = 105 \times 10^6.$$
$$(9.1 \times 10^{-9}) + (8.7 \times 10^{-9}) = 17.8 \times 10^{-9}.$$

To subtract one such expression from another, subtract the characteristics and preserve the same factor. Thus:

$$(54 \times 10^6) - (51 \times 10^6) = 3 \times 10^6.$$

If the factors have different exponents of the same sign the factor or factors of larger exponent must be reduced to the smaller exponent, by factoring. The characteristic of the expression thus treated is multiplied by the odd factor. This gives a new expression whose characteristic is added to the other, and the factor of smaller exponent is preserved for both.

Thus:

$$(5 \times 10^7) + (5 \times 10^9) = (5 \times 10^7) + (5 \times 100 \times 10^7) = 505 \times 10^7.$$

The same applies to subtraction. Thus:

$$(5 \times 10^9) - (5 \times 10^7) = (5 \times 100 \times 10^7) - (5 \times 10^7) = 495 \times 10^7.$$

If the factors differ in sign, it is generally best to leave the addition or subtraction to be simply ex-

pressed. However by following the above rule it can be done. Thus:

Add 5×10^{-2} and 5×10^{3}.

$5 \times 10^{3} = 5 \times 10^{5} \times 10^{-2}$: $(5 \times 10^{5} \times 10^{-2}) + (5 \times 10^{-2}) = 500005 \times 10^{-2}$. This may be reduced to a fraction $\frac{500005}{100} = 5000.05$.

To multiply add the exponents of the factors, for the new factor, and multiply the characteristics for a new characteristic. The exponents must be added algebraically: that is, if of different signs the numerically smaller one is subtracted from the other one, its sign is given the new exponent.

Thus:

$$(25 \times 10^{6}) \times (9 \times 10^{8}) = 225 \times 10^{14}.$$
$$(29 \times 10^{-8}) \times (11 \times 10^{7}) = 319 \times 10^{-1}.$$
$$(9 \times 10^{8}) \times (98 \times 10^{-2}) = 882 \times 10^{6}.$$

To divide, subtract (algebraically) the exponent of the divisor from that of the dividend for the exponent of the new factor, and divide the characteristics one by the other for the new characteristic. Algebraic subtraction is effected by changing the sign of the subtrahend, subtracting the numerically smaller number from the larger, and giving the result the sign of the larger number. (Thus to subtract 7 from 5 proceed thus: $5 - 7 = -2$.)

Thus:

$$(25 \times 10^{6}) \div (5 \times 10^{8}) = 5 \times 10^{-2}$$
$$(28 \times 10^{-8}) \div (5 \times 10^{3}) = 5.6 \times 10^{-11}.$$

TABLES.

I.—EQUIVALENTS OF UNITS OF LENGTH.

	Millimeter	Centim't'r	Meter	Kilometer	Mil	Inch	Foot	Yard	Mile (Statute)	Mile (Geog'h'l)
Millimeter	1	.01	.001	.000001	39.37079	.039371	.003281	.001094	.0000006	.0000007
Centimeter	10	1	.1	.00001	393.7079	.3937079	.032809	.010936	.0000062	.000007
Meter	1000	100	1	.001	39,370.79	39.37079	3.28090	1.09363	.000621	.000716
Kilometer	1,000,000	100,000	1000	1		39,370.79	3280.899	1093.633	.621382	.716330
Mil	.025399	.0025399	.0000254		1	.001	.000083	.000028		
Inch	25.3994	2.53994	.025399	.0000254	1000	1	.083333	.027777	.0000158	.000015
Foot	304.7945	30.47945	.304795	.0003048	12000	12	1	.333333	.000189	.000164
Yard	914.3835	91.43835	.914384	.0009144	36000	36	3	1	.000568	.000493
Mile (Statute)		160,931.4	1,609.314	1.609314		63,360	5280	1760	1	.868382
Mile (Geograph.)		185,329	1853.29	1.85329		72,963.2	6080.27	2026.76	1.1516	1

II.—EQUIVALENTS OF UNITS OF AREA.

	Square Millimeter	Square Centimet'r	Circular Mil.	Square Mil.	Square Inch.	Square Foot.
Square Millimeter	1	0.10	1973.6	1550.1	.00155	.0000108
Square Centimeter	100	1	197,361	155,007	.155007	.001076
Circular Mil.	.000507	.0000051	1	.78540	$8X10^{-7}$	
Square Mil.	.000645	.0000065	1.2733	1	.000001	
Square Inch	645.132	6.451	1,273,238	1,000,000	1	.006944
Square Foot	92,898.9	928.989			144	1

III.—EQUIVALENTS OF UNITS OF VOLUME.

	Cubic Inch	Fluid Ounce	Gallon	Cubic Foot	Cubic Yard	Cu. Centimeter	Liter	Cubic Meter
Cubic Inch	1	.554112	.004329	.000578		16.3862	.016386	
Fluid Oz.	1.80469	1	.007812	.001044		29.5720	.029572	
Gallon	231	128	1	.133681	.00495	3785.21	3.78521	.003785
Cubic Ft.	1728	957.506	7.48052	1	.037037	28315.3	28.3153	.028315
Cubic Yd.	46,656	25,852.6	201.974	27	1	764,505	764.505	.764505
Cu. Centi	.061027	.033816	.000264	.000035		1	.001	.000001
Liter	61.027	33.8160	.264189	.035317		1000	1	.001
Cu. Meter	61027	33816	264.189	35.3169	1.3080		1000	1

IV.—EQUIVALENTS OF UNITS OF WEIGHT.

	Grain.	Troy Ounce.	Pound Avs.	Ton.	Milli-gram.	Gram.	Kilo-gram.	Metric Ton.
Grain	1	.020833	.000143		64.799	.064799	.000065	
Troy Ounce	480	1	.068641		31,103.5	31.1035	.031104	
Pound Avs.	7,000	14.5833	1	.000447		453.593	.453593	.000454
Ton		32,666.6	2240	1			.001016	1.01605
Milligram	.015432	.000032	.000002		1	.001	.000001	
Gram	15.4323	.032151	.002205		1000	1	.001	
Kilogram	15,432.3	32.1507	2.20462	.000984	1,000,000	1000	1	.001
Metric Ton		32,150.7	2204.62	.98421		1,000,000	1000	1

V.—EQUIVALENTS OF UNITS

	Erg.	Meg-erg.	Gram-degree C.	Kilogram-degree C.	Pound-degree C.	Pound-degree F.
Erg.	1	.000001				
Meg.-erg.	1,000,000	1	.024068	.000024	.000053	.000095
Gram-degree C.		41.5487	1	.001	.002205	.003968
Kilogram-degree C.		41,548.7	1000	1	2.2046	3.9683
Pound-degree C.		18,846.5	453.59	.45359	1	1.8
Pound-degree F.		10,470.1	251.995	.251995	.555556	1
Watt-Second.	10^7	10	.24068	.000241	.000531	.000955
Gram-centimeter.	981	.000981	.0000235			
Kilogram-meter.	98.1×10^6	98.1	2.36108	.002361	.005205	.009370
Foot-Pound.		13.5626	.326425	.000326	.000720	.001295
Horse-Power-Sec. English.		7459.43	179.486	.179486	.3957	.71243
Horse-Power-Sec. Metric.		7357.5	177.075	177.075	.390375	.70275

OF ENERGY AND WORK.

Watt-Second.	Gram-Centim'tr.	Kilogram-meter.	Foot-Pound.	Horse-power-second English.	Horse-power-second Metric.	
10^{-7}	.001019					Erg.
.1	1019.87	.010194	.073734	.000134	.000136	Meg-erg.
4.15487	42,358.5	.423535	3.06355	.00557	.005647	Gram-degree C.
4154.87		423.535	3063.55	5.57	5.64703	Kilogram-degree C.
1884.65		192.114	1389.6	2.52653	2.56149	Pound-degree C.
1047.03		106.730	772	1.40364	1.42305	Pound-degree F.
1	10,193.7	.101937	.737337	.0013406	.0013592	Watt-Second.
.000098	1	.00001	.000072			Gram-Centimeter.
9.81	100,000	1	7.23328	.013152	.013334	Kilogram-meter.
1.35626	13,825.3	.138253	1	.0018182	.001843	Foot-Pound.
745.943		76.0392	550	1	1.01383	Horse-Power-Sec. English.
735.75		75	542.496	.986356	1	Horse-Power-Sec. Metric.

VI.—TABLE OF SPECIFIC RESISTANCES IN MICROHMS AND OF COEFFICIENTS OF SPECIFIC RESISTANCES OF METALS.

	Specific Resistance. Microhms.	Coefficients of Sp. Res.		Specific Resistance. Microhms.	Coefficients of Sp. Res.
Annealed Silver.....	1.521	.9412	Annealed Nickel	12.60	7.7970
Hard Silver..........	1.652	1.0223	Compres'd Tin......	13.36	8.2678
Annealed Copper....	1.616	1.0000	" Lead	19.85	12.2834
Hard Copper	1.652	1.0223	" Antimony	35.90	22.2158
Annealed Gold... ..	2.081	1.2877	" Bismuth .	132.70	82.1170
Hard Gold......... .	2.118	1.3107	Liquid Mercury.....	99.74	61.7208
Annealed Aluminum..	2.945	1.8224	2 Silver, 1 Platinum.	24.66	15.2599
Compressed Zinc....	5.689	3.5204	German Silver	21.17	13.1002
Annealed Platinum..	9.158	5.6671	2 Gold, 1 Silver	10.99	6.8008
" Iron	9.825	6.0798			

SPECIFIC RESISTANCE OF SOLUTIONS AND LIQUIDS.

MATTHIESSEN AND OTHERS.

Names of Solutions.	Temperature Centigrade.	Temperature Fahrenheit.	Specific Resistance. Ohms.
Copper Sulphate, concentrated...........	9°	48.2°	29.82
" with an equal volume of water	"	"	46 54
" with three volumes of water	"	"	77.68
Common Salt, concentrated.............	18°	55.4°	5.93
" with an equal volume of water..	"	"	6.00
" with two volumes of water....	"	"	9.24
" with three volumes of water.	"	"	11.89
Zinc Sulphate, concentrated....	14°	57.2°	28.00
" with an equal volume of water..	"	"	22.75
" with two volumes of water.....	"	"	29.75
Sulphuric Acid, concentrated..................	14.8°	57.8°	5.32
" 50.5%, Specific Gravity 1.398...	14.5°	58.1°	1.086
" 29.6%, Specific Gravity 1.215...	12.8°	54.5°	.83
" 12% Specific Gravity 1.080...	12.8°	55.0°	1.368
Nitric Acid, Specific Gravity 1.36 (Blavier)	14°	57.2°	1.45
" " " "	24°	75.2°	1.22
Distilled Water, (Temp'ture unknown) (Pouillet)			932.

VII.—RELATIVE RESISTANCE AND CONDUCTANCE OF PURE COPPER AT DIFFERENT TEMPERATURES.

MATTHIESSEN.

Temperature Centigrade.	Temperature Fahrenheit.	Relative Resistance.	Relative Conductance	Temperature Centigrade.	Temperature Fahrenheit.	Relative Resistance.	Relative Conductance
0°	32°	1.	1.	16°	60.8°	1.06168	.9419
1	33.8	1.00381	.99620	17	62.6	1.06563	.93841
2	35.6	1.00756	.9925	18	64.4	1.06959	.93494
3	37.4	1.01135	.98878	19	66.2	1.07356	.93148
4	39.2	1.01515	.98508	20	68.	1.07754	.92804
5	41	1.01896	.98139	21	69.8	1.08152	.92462
6	42.8	1.0228	.97771	22	71.6	1.08553	.92120
7	44.6	1.02663	.97406	23	73.4	1.08954	.91782
8	46.4	1.03048	.97042	24	75.2	1.09356	.91445
9	48.2	1.03435	.96679	25	77.	1.09759	.9111
10	50	1.03822	.96319	26	78.8	1.10162	.90776
11	51.8	1.04210	.95960	27	80.6	1.10567	.90443
12	53.6	1.04599	.95603	28	82.4	1.10972	.90113
13	55.4	1.0499	.95247	29	84.2	1.11382	.89784
14	57.2	1.05381	.94893	30	86.	1.11785	.89457
15	59	1.05774	.94541				

VIII.—AMERICAN WIRE GAUGE TABLE.

Properties of Copper Wire: Specific Gravity, 8.878; Specific Conductivity, 1.765 at 75° F.

Gauge Number	Size.		Weight and Length.			Resistance.			Carrying Capacity, 2,000 Amperes p.sq.in. section. Amperes
	Diameter in Mils.	Square of Diameter or circular Mils.	Grains per Foot.	Po'nds per 1000 Feet.	Feet per Pound.	Ohms per 1000 Feet.	Feet per Ohms.	Ohms per Pound.	
0000	460.000	211600.0	4477.2	639.60	1.564	.051	19929.7	.0000785	430
000	409.640	167804.9	3550.5	507.22	1.971	.063	15804.9	.000125	262
00	364.800	133079.0	2815.8	402.25	2.486	.080	12534.2	.000198	208
0	324.950	105592.5	2236.2	319.17	3.133	.101	9945.3	.000315	165
1	289.300	83694.49	1770.9	252.98	3.952	.127	7882.8	.000501	130
2	257.630	66373.22	1404.4	200.63	4.994	.160	6251.4	.000799	103
3	229.420	52633.53	1113.6	159.09	6.285	.202	4957.3	.001268	81
4	204.310	41742.57	883.2	126.17	7.925	.254	3931.6	.002016	65
5	181.940	33102.16	700.4	100.05	9.995	.321	3117.8	.003206	52
6	162.020	26250.48	555.4	79.34	12.604	.404	2472.4	.005098	41
7	144.280	20816.72	440.4	62.92	15.893	.509	1960.6	.008106	32
8	128.490	16509.68	349.3	49.90	20.040	.643	1555.0	.01289	26
9	114.430	13094.22	277.1	39.58	25.265	.811	1233.3	.02048	20
10	101.890	10381.57	219.7	31.38	31.867	1.023	977.8	.03259	16
11	90.742	8234.11	174.2	24.89	40.176	1.289	775.5	.05181	13
12	80.808	6529.93	138.2	19.74	50.659	1.626	615.02	.08237	10.2
13	71.961	5178.39	109.6	15.65	63.898	2.048	488.25	.13087	8.1
14	64.084	4106.75	86.87	12.41	80.580	2.585	386.80	.20830	6.4
15	57.068	3256.76	68.88	9.84	101.626	3.177	306.74	.33133	5.1
16	50.820	2582.67	54.67	7.81	128.041	4.582	243.25	.52638	4.0
17	45.257	2048.19	43.33	6.19	161.551	5.183	192.91	.83744	3.2
18	40.303	1624.33	34.37	4.91	203.666	6.536	152.99	1.3312	2.5
19	35.390	1252.45	26.50	3.786	264.136	8.477	117.96	2.2392	1.96
20	31.961	1021.51	21.60	3.086	324.045	10.394	96.21	3.3438	1.60
21	28.462	810.09	17.14	2.448	408.497	13.106	76.30	5.3539	1.28
22	25.347	642.47	13.59	1.942	514.933	16.525	60.51	8.5099	1.08
23	22.571	509.45	10.77	1.539	649.773	20.842	47.98	13.334	.80
24	20.100	404.01	8.55	1.221	819.001	26.284	38.05	21.524	.63
25	17.900	320.41	6.77	.967	1034.126	33.135	30.18	34.298	.50
26	15.940	254.08	5.38	.768	1302.083	41.789	23.93	54.410	.40
27	14.195	201.49	4.26	.608	1644.737	52.687	18.98	86.657	.31
28	12.641	159.79	3.39	.484	2066.116	66.445	15.05	137.283	.25
29	11.257	126.72	2.69	.384	2604.167	83.752	11.94	218.104	.20
30	10.025	100.50	2.11	.302	3311.258	105.641	9.466	349.805	.16
31	8.928	79.71	1.67	.239	4184.100	133.191	7.508	557.286	.13
32	7.950	63.20	1.33	.190	5263.158	168.011	5.952	884.267	.098
33	7.080	50.13	1.06	.151	6622.517	211.820	4.721	1402.78	.078
34	6.304	39.74	.847	.121	8264.463	267.165	3.743	2207.98	.062
35	5.614	31.52	.658	.094	10638.30	336.81	2.969	3583.12	.049
36	5.000	25.00	.525	.075	13333.33	424.65	2.355	5661.71	.039
37	4.453	19.83	.420	.060	16666.66	535.33	1.868	8922.20	.031
38	3.965	15.72	.315	.045	22222.22	675.22	1.481	15000.5	.025
39	3.531	12.47	.266	.038	26315.79	851.789	1.174	22415.5	.020
40	3.144	9.88	.210	.030	33333.33	1074.11	.931	35803.8	.015

IX.—CHEMICAL AND THERMO-CHEMICAL EQUIVALENTS.

Formation of Oxides.

Name of Compound.	Formula.	Valency.	Chemical Equivalents.	Combining Weights.	Thermo-Chemical Equivalents.
Water...	$H^2 O$	II	18	9	34.5
Iron Protoxide	Fe O	II	72	36	34.5
Iron Sesquioxide	$Fe^2 O^3$	III	160	53.3	31.9x3
Zinc Oxide	Zn O	II	81	40.5	43.2
Copper Oxide	Cu O	II	79.4	39.7	19.2
Mercury Oxide	Hg O	II	216	108	15.5

Formation of Salts.

Name of Base.	Valency.		Nitrates	Sulphates	Chlorides	Cyanides.
Iron	II	Formula.	$Fe (NO^3)^2$	$Fe SO^4$	$Fe Cl^2$	$Fe Cy^2$
		Chemical Equivalents	180	136	127	112
		Combining Weights	90	68	63.5	66
		Thermo-Chemical Equiv'lts	13.9	12.5	50	3.2
Zinc	II	Formula	$Zn (NO^3)^2$	$Zn SO^4$	$Zn Cl^2$	$Zn Cy^2$
		Chemical Equivalents	189	161	136	117
		Combining Weights	94.5	80.5	68	58.5
		Thermo-Chemical Equiv'lts	9.8	11.7	56.4	7.3
Copper	II	Formula.	$Cu (NO^3)^2$	$Cu SO^4$	$Cu Cl^2$	$Cu Cy^2$
		Chemical Equivalents	187.4	159.4	134.4	125.4
		Combining Weights	93.7	79.7	67.2	62.7
		Thermo-Chemical Equiv'lts	7.5	9.2	31.3	7.3
Mercury	II	Formula.	$Hg(NO^3)^2$	$Hg SO^4$	$Hg Cl^2$	$Hg Cy^2$
		Chemical Equivalents	324	280	271	252
		Combining Weights	162	140	135.5	126
		Thermo-Chemical Equiv'lts	7.5	9.2	9.45	15.5

X.—CHEMICAL AND ELECTRO-CHEMICAL EQUIVALENTS.

Name	Symbols	Valencies	Chemical Equivalents	Combining Weights	Electro-Chemical Equivalents
Hydrogen	H	I	1	1	.0105
Gold	Au	III	196.6	65.5	.6877
Silver	Ag	I	108	108	1.134
Copper (Cupric).......	$Cu_{\prime\prime}$	II	63	31.5	.3307
Mercury (Mercuric)...	$Hg_{\prime\prime}$	II	200	100	1.05
" (Mercurous)..	$Hg_{\prime}$	I	200	200	2.10
Iron (ferric)......... ..	$Fe_{\prime\prime\prime}$	III	56	18.7	.1964
" (ferrous)	$Fe_{\prime\prime}$	II	56	28	.294
Nickel	Ni	II	59	29.5	.3098
Zinc	Zn	II	65	32.5	.3413
Lead	Pb	II	207	103.5	1.0868
Oxygen	O	II	16	8	.84
Chlorine..............	Cl	I	35.5	35.5	.3728

XI.—MAGNETIZATION AND MAGNETIC TRACTION.

B Lines per sq. cm.	$B_{\prime\prime}$ Lines per sq. in.	Dynes per sq. centim.	Grammes per sq. centim.	Kilogrs. per sq. centim.	Pounds per sq. inch.
1,000	6,450	39,790	40.56	.0456	.577
2,000	12,900	159,200	162.3	.1623	2.308
3,000	19,350	358,100	365.1	.3651	5.190
4,000	25,800	636,600	648.9	.6489	9.228
5,000	32,250	994,700	1,014	1.014	14.39
6,000	38,700	1,432,000	1,460	1.460	20.75
7,000	45,150	1,950,000	1,987	1.987	28.26
8,000	51,600	2,547,000	2,596	2.596	36.95
9,000	58,050	3,223,000	3,286	3.286	46.72
10,000	64,500	3,979,000	4,056	4.056	57.68
11,000	70,950	4,815,000	4,907	4.907	69.77
12,000	77,400	5,730,000	5,841	5.841	83.07
13,000	83,850	6,725,000	6,855	6.855	97.47
14,000	90,300	7,800,000	7,550	7.550	113.1
15,000	96,750	8,953,000	9,124	9.124	129.7
16,000	103,200	10,170,000	10,390	10.39	147.7
17,000	109,650	11,500,000	11,720	11.72	166.6
18,000	116,100	12,890,000	13,140	13.14	186.8
19,000	122,550	14,630,000	14,630	14.63	208.1
20,000	129,000	15,920,000	16,230	16.23	230.8

XII.—PERMEABILITY OF WROUGHT AND CAST IRON.

SQUARE CENTIMETER MEASUREMENT.

Annealed Wrought Iron.			Gray Cast Iron.		
B	μ	H	B	μ	H
5,000	3,000	1.66	4,000	800	5
9,000	2,250	4	5,000	500	10
10,000	2,000	5	6,000	279	21.5
11,000	1,692	6.5	7,000	133	42
12,000	1,412	8.5	8,000	100	80
13,000	1,083	12	9,000	71	127
14,000	823	17	10,000	53	188
15,000	526	28.5	11,000	37	292
16,000	320	50	....	....	....
17,000	161	105	....	....	....
18,000	90	200	....	....	....
19,000	54	350	....	....	....
20,000	30	666	....	....	....

SQUARE INCH MEASUREMENT.

Annealed Wrought Iron.			Gray Cast Iron.		
B''	μ.	H''	B''	μ.	H''
30,000	4,650	6.5	25.000	763	32.7
40,000	3,877	10.3	30,000	756	39.7
50,000	3,031	16.5	40,000	253	155
60,000	2,159	27.8	50,000	114	439
70 000	1,921	36.4	60,000	74	807
80,000	1,409	56.8	70,000	40	1,480
90,000	907	99.2	....	....	....
100,000	408	245	....	....	....
110,000	166	664	....	....	....
120,000	76	1,581	....	....	....
130,000	35	3,714	....	....	....
140,000	27	5,185	....	....	....

PERMEABILITY OF SOFT CHARCOAL WROUGHT IRON.

(SHELFORD BIDWELL.)

SQUARE CENTIMETER MEASURE.

B	μ	H
7,890	1899.1	3.9
11,550	1121.4	10.3
15,460	386.4	40
17,330	150.7	115
18,470	88.8	208
19,330	45.3	427
19,820	33.9	585

SQUARE INCH MEASUREMENT.

$B_{\prime\prime}$	μ.	$H_{\prime\prime}$
47,414	1897	25.0
74,104	1122	66.1
99,191	388	256
111,189	150	738
118,504	88.8	1335
124,021	45.3	2740
127,165	33.9	3753

B — Magnetic Flux.
H — Magnetizing Force.
} Both in lines of force.

$\mu = \frac{B}{H}$ the Permeability or multiplying power of the core.

XIII.—MAGNETIC RELUCTANCE OF AIR BETWEEN TWO PARALLEL CYLINDERS OF IRON.

$\frac{b}{p}$ Ratio of least distance apart to circumference.	Centimeter Units.		Inch Units.	
0.1	.1954	5.1055	0.0771	12.968
0.2	.2707	3.6917	0.1066	9.377
0.3	.3251	3.0768	0.1280	7.815
0.4	.3683	2.7158	0.1450	6.897
0.5	.4046	2.4716	0.1593	6.278
0.6	.4361	2.2933	0.1717	5.825
0.8	.4887	2.0465	0.1924	5.198
1.0	.5316	1.8807	0.2093	4.777
1.2	.5684	1.7996	0.2238	4.571
1.4	.6007	1 6645	0.2365	4.228
1.6	.6289	1.5902	0.2476	4.039
1.8	.6541	1.5287	0.2575	3.883
2.0	.6774	1.4764	0.2667	3.750
4.0	.8357	1.1968	0.3290	3.040
6.0	.9319	1.0732	0.3669	2.726
8.0	1.0047	.9953	0.3955	2.528
10.0	1.0544	.9484	0.4151	2.409

In this table in columns 2 and 3 the Unit length of a cylinder is taken as 1 centimeter; in columns 4 and 5 as 1 inch. p = circumference of cylinder b = shortest distance apart.

XIV.—TABLE OF 6TH ROOTS.

Number	Sixth Root	Number	Sixth Root	Number	Sixth Root	Number	Sixth Root
$\frac{1}{9}$	.69355	$\frac{3}{4}$	.95320	$1\frac{1}{9}$	1.0177	$1\frac{3}{4}$	1.0978
$\frac{1}{8}$	.70717	$\frac{4}{5}$	.96350	$1\frac{1}{8}$	1.0192	$1\frac{4}{5}$	1.1019
$\frac{1}{7}$	.72306	$\frac{5}{6}$	.97006	$1\frac{1}{7}$	1.0226	$1\frac{5}{6}$	1.1063
$\frac{1}{6}$	.74185	$\frac{6}{7}$	.97463	$1\frac{1}{6}$	1.0260	$1\frac{6}{7}$	1.1087
$\frac{1}{5}$	.76478	$\frac{7}{8}$	.97798	$1\frac{1}{5}$	1.0308	$1\frac{7}{8}$	1.1107
$\frac{1}{4}$	.79370	$\frac{8}{9}$	.98055	$1\frac{1}{4}$	1.0379	$1\frac{8}{9}$	1.1119
$\frac{1}{3}$	.83268	$\frac{9}{10}$	.98258	$1\frac{1}{3}$	1.0491	$1\frac{9}{10}$	1.1129
$\frac{1}{2}$	.89090			$1\frac{1}{2}$	1.0699	2	1.1237
$\frac{2}{3}$	.93462			$1\frac{2}{3}$	1.0888		

XV.—STANDARD AND BIRMINGHAM WIRE GAUGES.

Standard.			Birmingham.		
Number of Gauge.	Diameter in Mils.	Square of Diameter or Circ'l'r Mils.	Number of Gauge.	Diameter in Mils.	Square of Diameter or Circ'l'r Mils.
0000000	500	250000	0000	454	206116
000000	464	215296	000	425	180625
00000	432	186824	00	380	144400
0000	400	160000	0	340	115600
000	372	138384	1	300	90000
00	348	121104	2	284	80656
0	324	104976	3	259	67081
1	300	90000	4	238	56644
2	276	76176	5	220	48400
3	252	63504	6	203	41209
4	232	53824	7	180	32400
5	212	44944	8	165	27225
6	192	36864	9	148	21904
7	176	30976	10	134	17956
8	160	25600	11	120	14400
9	144	20736	12	109	11881
10	128	16384	13	095	9025
11	116	13456	14	083	6889
12	104	10816	15	072	5184
13	092	8464	16	065	4225
14	080	6400	17	058	3364
15	072	5184	18	049	2401
16	064	4096	19	042	1764
17	056	3136	20	035	1225
18	048	2304	21	032	1024
19	040	1600	22	028	784
20	036	1296	23	025	625
21	032	1024	24	022	484
22	028	784	25	020	400
23	024	576	26	018	324
24	022	484			
25	020	400			
26	018	324			

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